VACUUM INSTABILITY AND MAJORANA SPINORS OF THE BLACK CIRCLE

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ABSTRACT. Paper 17 derived the complete QNM condition for $l \geq 1$ under Candidate (a) and found growing modes at $l = 0, 1$ with near-universal timescale $\tau_{\text{grow}} \approx 1.36\, r_s/c$. This paper computes the resulting one-loop effective action and takes the first steps toward the Majorana spinor sector.

OP 18.4 closed *[Conjecture]*. The one-loop imaginary part is revised using the corrected QNM spectrum (Papers 15–17):

$$\mathrm{Im}(W_{\text{1-loop}}) \approx 70.848 - 31.009\, m_4^2 \quad [\mathrm{r}_s^{-1}], \quad m_4^2 \in (0, 0.256),$$

where m_4^2 is the Level-4 mass under the cavity bound. The sign is reversed relative to Paper 14's $-74.800\, r_s^{-1}$ (which used artifact QNMs with $\mathrm{Im}(\omega^*) < 0$ only). The positive value, controlled by the growing modes, signals a metastable vacuum: $\mathrm{Im}(W) > 0$ implies $e^{iW} = e^{i\ \mathrm{Re}(W)} \cdot e^{-\ \mathrm{Im}(W)} \to 0$ for stable configurations, consistent with the Black Circle as a condensate rather than a ground state.

OP 18.6 partially closed *[Conjecture]*. The quasi-universal growth timescale is explained by $|A_{\text{part}}(\omega_l^*; l, 0)| \approx 0.479$ for both $l = 0$ and $l = 1$ (0.25% difference), giving a common modulus balance $|A_{\text{part}} \cdot 2^{i\Omega}| \approx 0.371\, r_s^{-1}$. The algebraic origin remains OP 18.6 *[Open]*.

Slope correction *[Derived]*. The mass slope for the $l = 0$ growing mode, $d\kappa_{\text{grow}}/dm^2\big|_{l=0} = +0.14010$ (positive), is opposite in sign to the $l = 1$ slope -0.2688. The corrected one-loop slope is $d\sigma/dm^2 = -0.7383\, r_s^{-1}$.

OP 12.7 partially closed. The Frobenius–Schur indicator of the spinorial irrep 4_s in $\mathrm{GL}(2,3) \cong 2S_4$ is $\mathrm{FS}(4_s) = +1$ *[Derived]*, computed directly from the character table. This establishes: 4_s is real; charge conjugation C exists with $C^2 = +1$; the Majorana condition $\psi = C\psi$ is representation-theoretically consistent. The remaining obstacle is OS2 *[Open]* (OP 18.7): reflection positivity on the compact de Sitter interior.

New open problems. OP 18.7 (OS2 for compact dS_4: does the Lüscher–Mack theorem generalize?) and OP 18.8 (four 4_s modes vs. three Standard Model generations, with sterile neutrino as preferred resolution) are stated precisely.

Epistemic key. *[Derived]* — follows from Papers 1–17 and established mathematics. *[Conjecture]* — precise statement; additional physical assumptions stated explicitly. *[Speculation]* — structurally motivated; not currently falsifiable. *[Open]* — obstacle stated precisely; result not yet obtained.

1. Introduction

1.1. IIB-II and testability.
The physical core of the series is IIB-II (Paper 2, *[Conjecture]*): the quadratic norm N is the unique structure preserved at every Cayley–Dickson doubling step, and gravity is associated with N rather than with any multiplication structure. Paper 2 states explicitly: *"IIB-II becomes testable if a QFT over $\mathcal{A}_4$ is constructed."* Paper 18 is the first step toward this testability: $\mathrm{Im}(W_{\text{1-loop}})$ is the first quantum correction to the $\mathcal{A}_4$ field-theory action.

IIB-III (Paper 2, *[Conjecture]*): degrees of freedom in $\mathcal{A}_4 \setminus \mathcal{A}_3$ are gauge-invisible (no Standard Model quantum numbers) and gravitationally active. The 4 spinorial modes of $\ker(L_a)$ in Paper 12

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live in this sector; their Majorana character (OP 12.7) would make them dark-matter candidates under IIB-III and elementary-fermion candidates under Conjecture A simultaneously.

1.2. **What Papers 15–17 established.** Paper 15: exact QNM condition $D_{\rm in}^{\rm true}(l=0)=K_1\cdot 2^{i\Omega}$; no damped $l=0$ modes at $m^2=0$; growing mode $\kappa_{\rm grow}=0.73472367$, $\tau_{\rm grow}(l=0)=1.361\,r_s/c$. Type II $l=0$ curve with $m^2_{\rm thr}=0.4605600022$ *[Derived]*. Paper 16: for $l\geq 1$, limit-circle singularity at $r=0$; $L_w[w_{\rm phys}]=2l(l+1)/z$; Kummer coefficients $\tilde{K}_1^{(\varrho_j,l)}$. Paper 17: $\phi_j^{\rm hom}\sim r^{-1/2}\to\infty$ (singular); Candidate (a) gives $C_1=C_2=0$; complete QNM condition $H_{\rm BC}^{\rm full}=[A_{\rm part}+C_1\tilde{K}_1^{(\varrho_1)}+C_2\tilde{K}_1^{(\varrho_2)}]\cdot 2^{i\Omega}-2^{i\omega/2}e^{i\omega}=0$; growing mode $l=1$: $\omega^*=1.6449+0.7372i$, $\tau_{\rm grow}(l=1)=1.357\,r_s/c$ *[Conjecture]* (IIB-I/II).

1.3. **Organisation.** Section 2: QNM spectrum for $l=1,2$ at $m^2\neq 0$ (OP 18.3). Section 3: $\mathrm{Im}(W)$ revised (OP 18.4) and slope correction. Section 4: universality of $\tau_{\rm grow}$ (OP 18.6). Section 5: FS indicator and Majorana sector (OP 12.7). Section 6: critical collapse radius *[Speculation]*. Section 7: open problems.

2. QNM spectrum for $l=1,2$, $m^2\neq 0$ (OP 18.3)

Proposition 2.1 (OP 18.3, *[Conjecture]* (Candidate (a), IIB-I/II))**.** *The $l=1$ growing mode is independently confirmed at $m^2=0$: the Wronskian code with $z_{\rm match}=0.97$, $N=5000$ terms, mpmath at 50 d.p. gives $\tau=1.359\,r_s/c$, agreeing with Paper 17's $\tau=1.357\,r_s/c$ to 0.15%.*

Complete $m^2=0$ spectrum (truncated at $l\leq 2$, contributions from $l\geq 3$ neglected; see Remark 2.2):

l	*Mode*	ω^*	τ
0	*growing*	$+0.73472367\,i$	$1.361\,r_s/c$
1	*growing*	$1.6449+0.7372\,i$	$1.357\,r_s/c$
1	*damped*	$1.2979-0.0076\,i$	$131\,r_s/c$
2	*damped*	$1.9252-0.4160\,i$	$2.4\,r_s/c$

Mass slope [Derived] *for $l=0$ (analytic IFT, §3): $d\kappa_{\rm grow}/dm^2\big|_{l=0}=+0.14010$. Mass slope* [Conjecture] *for $l=1$ (Paper 17, §6.4): -0.2688. Slopes for damped modes estimated numerically from drifted zeros: $|d(\mathrm{Im}\,\omega^*)/dm^2|\approx 0.009$ for both $l=1,2$ (uncertainty $\sim 50\%$; contribution to $d\sigma/dm^2$ is 9% of the dominant term).*

Remark 2.2 (*[Derived]*)**.** The formula $\sigma=\sum_l(2l+1)\,\mathrm{Im}(\omega_l^*)$ is truncated at $l=2$. Contributions from $l\geq 3$ require QNM computations not performed in this series; their omission is part of the *[Conjecture]* label on $\mathrm{Im}(W)$. For a Schwarzschild background, higher-l QNM contributions are exponentially suppressed; we assume the same ordering holds here.

3. One-loop imaginary part (OP 18.4)

3.1. Slope for the $l=0$ growing mode.

Proposition 3.1 (*[Derived]*)**.** *For the $l=0$ growing mode with general m^2, the QNM condition is $F(\kappa,m^2)\equiv 2^{(\kappa'+\kappa)/2-1}e^{\kappa}-(1+\kappa')=0$ where $\kappa'=\sqrt{\kappa^2+m^2}$ (Paper 15). By the implicit function theorem at $(\kappa_{\rm grow},m^2=0)$ where $\kappa'=\kappa$:*

$$\partial_\kappa F\big|_{m^2=0}=(1+\kappa_{\rm grow})(\ln 2+1)-1=1.93714, \tag{1}$$

$$\partial_{m^2}F\big|_{m^2=0}=\frac{1}{2\kappa_{\rm grow}}\left[\frac{(1+\kappa_{\rm grow})\ln 2}{2}-1\right]=-0.27139, \tag{2}$$

$$\left.\frac{d\kappa_{\rm grow}}{dm^2}\right|_{m^2=0}=-\frac{\partial_{m^2}F}{\partial_\kappa F}=\frac{0.27139}{1.93714}=\boxed{+0.14010.} \tag{3}$$

Numerical verification: Newton iteration on $F = 0$ confirms $d\kappa/dm^2 \approx +0.140$ to $< 1\%$ at $m^2 = 0$.

The slope is ***positive for*** *$l = 0$ and* ***negative for*** *$l = 1$ (-0.2688, Paper 17): as m^2 increases, the $l = 0$ growing mode becomes faster (more unstable) while the $l = 1$ growing mode becomes slower.*

3.2. **Corrected one-loop formula.**

Theorem 3.2 (OP 18.4 closed, *[Conjecture]*)**.** *The Minkowski-convention one-loop imaginary part, $\mathrm{Im}(W_{1\text{-}loop}) = \sum_{fields} \sigma(m_i^2)$ with $\sigma = \sum_{l=0}^{2}(2l+1)\,\mathrm{Im}(\omega_l^*)$, evaluated at m^2=0 gives $\sigma_0 = 0.84343\,r_s^{-1}$.*

The corrected mass slope, incorporating $l = 0$ and $l = 1$ growing modes and estimated damped-mode contributions:

$$\frac{d\sigma}{dm^2} = 1\cdot(+0.14010) + 3\cdot(-0.26880) + 3\cdot(-0.009) + 5\cdot(-0.009) = -0.7383\,r_s^{-1}. \tag{4}$$

Using $\sum_i N_i m_i^2 = 42\,m_4^2$ (Paper 16, Proposition 6.1):

$$\boxed{\mathrm{Im}(W_{1\text{-}loop}) \approx 70.848 - 31.009\,m_4^2 \quad [r_s^{-1}], \quad m_4^2 \in (0,\, 0.256).} \tag{5}$$

Proof. $\mathrm{Im}(W) = 84\sigma_0 + (d\sigma/dm^2)\cdot\sum_i N_i m_i^2 = 84 \times 0.84343 + (-0.7383) \times 42\,m_4^2 = 70.848 - 31.009\,m_4^2$. □

m_4^2	$\mathrm{Im}(W)$
0.000	$+70.848\,r_s^{-1}$
0.064	$+68.863\,r_s^{-1}$
0.128	$+66.879\,r_s^{-1}$
0.192	$+64.894\,r_s^{-1}$
0.256	$+62.910\,r_s^{-1}$

Remark 3.3 (*[Conjecture]*)**. Sign convention and physical interpretation.** With the Minkowski convention $Z = \int \mathcal{D}\phi\, e^{iW}$: $e^{iW} = e^{i\,\mathrm{Re}(W)} \cdot e^{-\mathrm{Im}(W)}$. For $\mathrm{Im}(W) > 0$ the path-integral weight is exponentially suppressed relative to a stable vacuum, consistent with a metastable condensate. The positive value is controlled by the growing QNM modes ($\mathrm{Im}(\omega^*) > 0$) found in Papers 15–17, which were absent from Paper 14's spectrum. Comparison: Paper 14 found $\mathrm{Im}(W) = -74.800\,r_s^{-1}$ (negative) using artifact QNMs with $\mathrm{Im}(\omega^*) < 0$ only. Sign reversed; magnitude comparable ($|\mathrm{Im}(W)| \approx 63$–$71\,r_s^{-1}$).

Level	N	$m_i^2\big\rvert_{m_4^2=0.128}$	$\sigma(m_i^2)$	$N \times \sigma$
1 ($\mu = -4$)	7	-0.0640	0.8907	6.235
2 ($\mu = -2$)	14	0.0000	0.8434	11.808
3 ($\mu = 0$)	42	$+0.0640$	0.7962	33.440
4 ($\mu = +2$)	14	$+0.1280$	0.7490	10.485
5 ($\mu = +4$)	7	$+0.1920$	0.7017	4.912
Total	84			$66.879\,r_s^{-1}$

4. Universality of τ_{grow} (OP 18.6)

Proposition 4.1 (*[Conjecture]*, partial)**.** *The quasi-universal growing timescale, $\tau_{\text{grow}}(l = 0) = 1.361\,r_s/c$ and $\tau_{\text{grow}}(l = 1) = 1.357\,r_s/c$ (difference 0.33%), arises from a common modulus balance:*

$$|K_1^{l=0}(\omega_{l=0}^*)| = 0.479638, \quad |A_{\text{part}}^{l=1}(\omega_{l=1}^*)| = 0.478458 \quad (\textit{diff. } 0.25\%),$$
$$|2^{i\Omega(l=0)}| = 0.775199, \quad |2^{i\Omega(l=1)}| = 0.774537 \quad (\textit{diff. } 0.085\%).$$

Both satisfy the same modulus balance $|A_{\rm part}\cdot 2^{i\Omega}|\approx 0.371\,r_s^{-1}=f(\kappa_{\rm grow})$ *where* $f(\kappa)=2^{-\kappa/2}e^{-\kappa}$ *is independent of* l*.*

The mechanism is [Conjecture]*: at* $m^2=0$*, the growing mode frequency is controlled by the EF boundary condition at* $z=1$ *through* $2^{i\omega/2}e^{i\omega}$*, independently of* l*.*

Remark 4.2. The algebraic origin of $|K_1^{l=0}|\approx|A_{\rm part}^{l=1}|$ (0.25% agreement) is declared as OP 18.6 *[Open]*. Whether $\tau_{\rm grow}(l)$ is universal for all $l\geq 0$ requires computation of the $l=2$ growing mode (OP 18.5).

5. Frobenius–Schur indicator and Majorana spinors (OP 12.7)

5.1. **Setup.** Paper 12 *[Derived]*: the 4 spinorial modes in $\ker(L_a)\subset$ Level-3 transform under $\mathrm{GL}(2,3)\cong 2S_4$ in the 4-dimensional irrep 4_s. The CAR algebra and $H_{\rm BC}=F_{\rm sym}(\mathbb{C}^{80})\otimes\wedge^\bullet(\mathbb{C}^4)$ hold. OP 12.7 asks whether the PCT involution satisfies the Osterwalder–Schrader axioms, making 4_s a Majorana field.

5.2. **Frobenius–Schur indicator.**

Theorem 5.1 (*[Derived]*, GAP 4.15.1)**.** *The Frobenius–Schur indicator of* 4_s *in* $\mathrm{GL}(2,3)$ *is*

$$\mathrm{FS}(4_s)=\frac{1}{|\mathrm{GL}(2,3)|}\sum_{g\in\mathrm{GL}(2,3)}\chi_{4_s}(g^2)=\frac{48}{48}=+1. \tag{6}$$

Computation from the character table (Python; confirmed by GAP 4.15.1 [21] *on local hardware, Threadripper 2950X):*

Class	$\lvert C_i\rvert$	*Ord*	$g^2\to$	$\chi_{4_s}(g^2)$	$\lvert C_i\rvert\cdot\chi_{4_s}(g^2)$
$1A$	1	1	$1A$	4	+4
$2A$ *(central)*	1	2	$1A$	4	+4
$2B$	12	2	$1A$	4	+48
$3A$	8	3	$3A$	1	+8
$4A$	6	4	$2A$	−4	−24
$6A$	8	6	$3A$	1	+8
$8A$	6	8	$4A$	0	0
$8B$	6	8	$4A$	0	0
Σ	48				+48

Check: $\sum_i|C_i||\chi_{4_s}(C_i)|^2=1\cdot16+1\cdot16+12\cdot0+8\cdot1+6\cdot0+8\cdot1+6\cdot0+6\cdot0=48=|\mathrm{GL}(2,3)|$✓

Corollary 5.2 (*[Derived]*, GAP 4.15.1)**.**
(i) 4_s *admits a* $\mathrm{GL}(2,3)$*-invariant real structure:* $4_s\cong\overline{4_s}$*.*
(ii) Charge conjugation $C:4_s\to4_s$ *exists with* $C^2=+1$*.*
(iii) The Majorana condition $\psi=C\psi$ *is representation-theoretically consistent (no contradiction from the group theory alone).*

5.3. **Status of the Osterwalder–Schrader axioms.**

Axiom	Status	Reason
OS1 (Euclidean covariance)	*[Conjecture]* (likely)	Rotational symmetry of BC metric
OS2 (Reflection positivity)	*[Open]* OP 18.7	Compact dS_4 interior
OS3 (Symmetry)	*[Derived]*	CAR algebra, Paper 12

Remark 5.3 (*[Open]*, OP 18.7)**.** The standard OS2 theorem (Osterwalder–Schrader 1973/1975 [18, 19]) requires a half-space $\{x_0>0\}$ in Euclidean space. The Black Circle interior $r\in[0,r_s]$ is compact with boundary at $r=0$ (regular singular point) and $r=r_s$ (EF junction). The standard

argument fails because the half-space is replaced by a compact domain. A generalization (possibly via the Lüscher–Mack theorem [20] on compact domains, or via the self-adjoint extension at $r = 0$) is needed. If OS2 fails, PCT is not derivable from the Streater–Wightman reconstruction, and the Majorana interpretation of 4_s requires an alternative justification.

5.4. **IIB-III connection and the generation problem.** Under IIB-III (Paper 2, *[Conjecture]*): the 4 modes of 4_s live in $\mathcal{A}_4 \setminus \mathcal{A}_3$ and are gauge-invisible but gravitationally active (dark-matter candidates). Combined with Corollary 5.2: if OS2 is confirmed, the 4_s sector consists of 4 Majorana dark-matter fermions.

Remark 5.4 (*[Open]*, OP 18.8)**.** The Standard Model has exactly 3 generations of fermions. The 4_s sector has 4 modes. Three resolutions:

(i) **(Preferred)** One mode is a sterile right-handed neutrino: 3 observed SM generations plus 1 unobserved. Falsifiable via neutrino oscillation experiments.
(ii) A symmetry-breaking mechanism projects $4_s \to 3$ modes in the low-energy limit. No such mechanism identified.
(iii) The 4-mode count is fundamental; the SM 3-generation count requires re-examination.

Option (i) is the preferred hypothesis for Paper 19 investigation.

6. Critical collapse radius

This entire section is *[Speculation]*.

During gravitational collapse, the Schwarzschild radius grows as $r_s(t) = 2GM(t)/c^2$. The Black Circle condensate C_0 requires time $\tau_{\text{grow}} \approx 1.36\, r_s/c$ to form (quasi-universal over $l = 0, 1$, Paper 17 + §4).

Critical radius. For a shell in free fall from R_0: $\tau_{\text{ff}} \approx (r_s/c) \cdot (R_0/r_s)^{3/2}$. Setting $\tau_{\text{ff}} = \tau_{\text{grow}}$:

$$\frac{R_c}{r_s} = (1.361)^{2/3} = e^{(2/3)\ln 1.361} \approx 1.228. \tag{7}$$

For $R_0/r_s > 1.228$: $\tau_{\text{ff}} > \tau_{\text{grow}}$, condensate can form. For $R_0/r_s < 1.228$: condensate cannot stabilise before crossing. Neutron stars have $R_{\text{NS}}/r_s \approx 2$–$4$; standard stellar collapse starts from $R_0/r_s \gg 1$.

The quasi-universal τ_{grow} implies that the 0.33% difference between $l = 0$ and $l = 1$ growing modes would produce a small correction to the standard Schwarzschild ringdown ratio, detectable only by next-generation gravitational-wave detectors (Einstein Telescope, Cosmic Explorer).

7. Open problems

Label	Statement	Status
OP 18.1	Full functional-analytic self-adjoint extension at $r = 0$, $l \geq 1$.	*[Open]*
OP 18.2	Non-trivial AMT BC: when do $(C_1, C_2) \neq (0, 0)$ arise from IIB-II?	*[Open]*
OP 18.3	QNMs for $l = 1, 2$, $m^2 \neq 0$: independent verification.	*[Open]* (partial)
OP 18.4	$\text{Im}(W)$ normalized with corrected $\|\chi_n\|^2$.	*[Open]* (OP 14.3)
OP 18.5	Off-axis zeros for $l = 1$ with $\text{Im}(\omega) > 1$.	*[Open]*
OP 18.6	Algebraic origin of $\lvert K_1^{l=0}\rvert \approx \lvert A_{\text{part}}^{l=1}\rvert$ (0.25%).	*[Open]*

Label	Statement	Status
OP 18.7	OS2 (reflection positivity) for spinors in compact dS_4: Lüscher–Mack generalization.	*[Open]*, new
OP 18.8	4_s has 4 modes; SM has 3 generations. Preferred: sterile neutrino.	*[Open]*, new
OP 14.3	Exact $\mathrm{Im}(W)$: inner product from S_{84} + zeta regularization.	*[Open]*
OP 12.1	Prove/disprove $\chi(r_s) = 0$ for spinorial Level-3 modes.	*[Open]*

8. Conclusions

OP 18.4 closed *[Conjecture]*. $\mathrm{Im}(W_{\text{1-loop}}) \approx 70.848 - 31.009\, m_4^2 \in (62.9, 70.9)\, r_s^{-1}$. Sign reversed relative to Paper 14's $-74.800\, r_s^{-1}$. The Black Circle is a metastable condensate, not a stable vacuum. This is the first quantum contribution toward the testability of IIB-II.

Slope correction *[Derived]*. $d\kappa_{\text{grow}}/dm^2\big|_{l=0} = +0.14010$ (positive, opposite to $l = 1$'s -0.2688). The corrected one-loop slope is $d\sigma/dm^2 = -0.7383\, r_s^{-1}$.

OP 18.6 partial *[Conjecture]*. τ_{grow} quasi-universality explained by $|A_{\text{part}} \cdot 2^{i\Omega}| \approx 0.371$ independent of l. Algebraic origin OP 18.6.

OP 12.7 partial *[Derived]* + *[Open]*. $\mathrm{FS}(4_s) = +1$ *[Derived]*: 4_s is real, $C^2 = +1$, Majorana condition consistent. OS2 is the remaining obstacle (OP 18.7).

IIB-III and generation problem. If OS2 is confirmed, 4_s gives 4 Majorana dark-matter fermions (IIB-III). Three-generation vs. four-mode: sterile neutrino preferred (OP 18.8).

Errata from previous papers

Paper 14, $\mathrm{Im}(W) = -74.800\, r_s^{-1}$ *[Derived]*: superseded by (5); sign reversed due to artifact QNM spectrum used in Papers 13–14.

Paper 14, $d\sigma/dm^2 = -1.0752$ *[Conjecture]*: corrected to -0.7383 (wrong $l = 0$ slope sign in draft v3; corrected by Proposition 3.1).

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Numerical computations in Python 3 (NumPy, SciPy, mpmath at 50 decimal places). $\mathrm{FS}(4_s) = +1$ independently verified with GAP 4.15.1 (Threadripper 2950X): unique 4-dimensional irrep of $\mathrm{GL}(2, 3)$ confirmed, full character table structure consistent with Paper 12. Computational assistance was provided by Claude (Anthropic) for algebraic verification, structural analysis, and proof scaffolding. The author takes full responsibility for any errors.

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