ONE-LOOP STABILITY AND BACKREACTION OF THE BLACK CIRCLE

GIOVANNI LEVRATTI

Abstract. Paper 18 established $\mathrm{Im}(W_{\text{1-loop}}) > 0$ — the Black Circle condensate C_0 is metastable — but left open whether C_0 is a local minimum or a saddle point of the one-loop effective potential, and whether the metastability is affected by the gravitational backreaction predicted by IIB-II.

This paper answers both questions.

Step 1 *[Derived]*: $V_{\text{classical}}(C) = 64\lambda C^4$; C_0 is a gravitational background (not a scalar-potential minimum), satisfying the Einstein equation but not the scalar EOM. The Hessian at C_0 has 7 negative eigenvalues $m_1^2 = -32\lambda C_0^2$ in the irr_7 sector, 14 zero eigenvalues in Level 2, and 63 positive eigenvalues in Levels 3–5 ($7 + 14 + 63 = 84$): a tachyonic saddle point.

Step 2 *[Derived]*: The Coleman–Weinberg one-loop mass correction in the irr_7 direction is $\delta m_1^2|_{\text{direct}} \approx +1.08\lambda^2 C_0^2$ (positive, partial stabilisation), insufficient to flip the sign of m_1^2 for $\lambda \ll 1$.

Step 3 *[Derived]*, cond. IIB-II: Under the self-consistent Einstein equation at one loop, the condensate shifts to $C_0^{(1)} = C_0(1+0.912\lambda)$. The gravitational backreaction dominates the direct correction by a factor of 54: $m_1^{2,\text{eff}} = -32\lambda C_0^2(1 + 1.79\lambda) < 0$. **Case (a): metastability is enhanced at one loop. IIB is perturbatively consistent and does not require reformulation.**

First IIB-II loop prediction *[Conjecture]*, cond. IIB-II: $\langle N\rangle_{\text{1-loop}}/C_0^2 \approx -8.41\lambda < 0$. Quantum fluctuations of the S_{84} fields reduce the expectation value of the gravitational correlate N below its classical value.

AMT compatibility *[Derived]*: the Analytical Mean Theorem $\mathbb{E}_v[N(av)] = N(a)$ is a classical algebraic identity unaffected by quantum vacuum corrections. GHY boundary corrections are subleading under the cavity bound.

Four open problems are stated: OP 20.1 (μ-independent case (a) via two-loop RGE), OP 20.2 (quantum GHY correction), OP 20.3 ($\delta G_{\text{eff}}/G$ from IIB-II), OP 20.4 (β-function and two-loop RGE, proposed for Paper 21).

Epistemic key. *[Derived]* — follows from Papers 1–19 and established mathematics. *[Conjecture]* — precise statement, assumptions explicit, depends on IIB-II. *[Speculation]* — structurally motivated, not falsifiable. *[Open]* — obstacle stated precisely.

Contents

Date: June 30, 2026.

2020 *Mathematics Subject Classification.* 17A35, 83C57, 81T20, 47B25, 81T15, 83C45, 83F05.

Key words and phrases. Cayley–Dickson algebras, sedenions, Black Circle, one-loop effective potential, Coleman–Weinberg, gravitational backreaction, IIB-II conjecture, norm-gravity, sedenionic condensate, metastability, Breitenlohner–Freedman bound, tachyonic sector, PSL(2,7), irr_7, background field method.

1. Introduction

1.1. **What Papers 1–19 established.** Papers 1–9 built the Cayley–Dickson zero-divisor algebraic foundation: Fano geometry, $\mathrm{PSL}(2,7)$, ZD zeta function, Reed–Muller CSS codes, Clifford representation, Fano duality, factorisation threshold, the $\mathcal{A}_3 \to \mathcal{A}_4$ loop threshold, and the unique quartic coupling λ of $\mathcal{A}_4$.

Paper 10 [10] constructed the 84-mode scalar action S_{84} on the static de Sitter interior of the Black Circle. Paper 2 [2] (IIB) formulated the IIB-II conjecture: $N = \sum_k \phi_k^2$ is the algebraic correlate of gravity. Paper 11 [11] performed canonical quantisation: CCR without modification, mass spectrum $\{m_i^2 = 16\lambda C_0^2(2+\mu_i)\}$ with multiplicities $\{7, 14, 42, 14, 7\}$, condensate $C_0^4 = 3/(2048\pi G^3 M^2 \lambda)$, BF bound $\lambda G M^2 < 27\pi/128$.

Papers 12–17 [12, 13, 14, 15, 16, 17] developed the QNM machinery: Frobenius obstruction, hypergeometric reduction, cavity bound $\lambda G M^2 < 2.14 \times 10^{-3}$, Kummer connection matrix, limit-circle obstacle, physical boundary condition at $r = 0$.

Paper 18 [18] established $\mathrm{Im}(W_{\text{1-loop}}) > 0$ via growing QNM modes ($\tau_{\text{grow}} \approx 1.36\, r_s/c$): the condensate is metastable. Paper 19 [19] identified four Majorana spinors among the Level-3 modes and derived $m_{4_s}^2 = 32\lambda C_0^2$ (equivalently $m_{4_s} = \sqrt{32\lambda}\, C_0$) *[Derived]*.

1.2. **The question this paper answers.** Paper 18 established that $\mathrm{Im}(W) > 0$ — the condensate is metastable. The open question: does $V''_{\text{eff}}(C_0) > 0$ (local minimum, IIB perturbatively consistent) or < 0 (saddle point, IIB requires recasting)?

The answer is richer than either option. C_0 is a *gravitational background* imposed by the Einstein equation, not a minimum of any scalar potential. The perturbative framework is the background field method. The answer has two parts: in the condensate direction (irr_1): $V''_{\text{eff}} > 0$ (stable); in the tachyonic sector (irr_7, 7 modes): $V''_{\text{eff}} < 0$ (saddle point), *enhanced at one loop by the gravitational backreaction.*

Under IIB-II, this is a prediction: the gravitational field N does not self-cure its own tachyonic sector.

1.3. **Organisation.** Section 2 collects the necessary background. Section 3 analyses the classical potential. Section 4 computes the one-loop mass correction. Section 5 derives the gravitational backreaction and the first IIB-II loop prediction. Section 6 discusses physical implications. Section 7 states four open problems. Section 8 concludes.

2. Preliminaries

2.1. **The action S_{84}.**

$$S_{84} = \int \sqrt{-g}\left[\sum_{k=1}^{84} \tfrac{1}{2}(\nabla\phi_k)^2 + 4\lambda \sum_{(A,B)\in \mathrm{ZD}} \phi_A^2\phi_B^2 + \tfrac{\xi}{2}R\sum_k N(\phi_k)\right] d^4x + S_{\mathrm{GHY}} \tag{1}$$

on the static de Sitter interior metric $f_{\mathrm{in}}(r) = r^2/r_s^2 - 1$ (Black Circle, Papers 2 and 10 [2, 10]). The ZD pair graph $\mathrm{ZD}^*(\mathcal{A}_4)$ is 4-regular on 84 vertices with 168 edges (Paper 11, Theorem 4.2 [11]). The coupling $\lambda \in \mathbb{R}$ is the unique quartic coupling of $\mathcal{A}_4$ (Paper 9, Corollary 3.20 [9]). The canonical commutation relations hold without modification (Paper 11, Proposition 3.2 [11]): S_{84} is an ordinary QFT of 84 real scalars.

The condensate is mixed (Paper 10, §7 [10]): 4 spinorial modes and their adjacent tensorial partners contribute exactly 16 active cross-pairs, yielding $V = 4\lambda \times 16 \times C_0^4 = 64\lambda C_0^4$.

2.2. **Mass spectrum and PSL(2,7) decomposition.** *[Derived]*, Paper 11 [11]: The Hessian of V_{int} at the condensate $\phi_k = C_0$ is $H_{kj} = 16\lambda C_0^2(2\delta_{kj} + A_{kj})$, where A is the adjacency matrix of the ZD graph. Its eigenvalues, organised by PSL(2,7) representation theory (Paper 11, Theorem 4.4–4.5):

$$m_i^2 = 16\lambda C_0^2(2+\mu_i) + \tfrac{12\xi}{r_s^2}. \tag{2}$$

Level	μ_i	Mult.	$\mathrm{irr}(\mathrm{PSL}(2,7))$	m_i^2 $(\xi = 0)$
5	$+4$	7	$\mathrm{irr}_1 \oplus \mathrm{irr}_3 \oplus \mathrm{irr}_3'$	$+96\lambda C_0^2$
4	$+2$	14	$2 \cdot \mathrm{irr}_7$	$+64\lambda C_0^2$
3	0	42	$3\,\mathrm{irr}_6 \oplus 3\,\mathrm{irr}_8$	$+32\lambda C_0^2$
2	-2	14	$\mathrm{irr}_6 \oplus \mathrm{irr}_8$	0
1	-4	7	irr_7	$-32\lambda C_0^2$

Mode decomposition *[Derived]*, Paper 19 [19]: $84 = 80_{\mathrm{boson}} + 4_{\mathrm{Majorana}}$. The four spinorial modes lie in $\ker(L_a)$ at Level 3 with $m_{4_s}^2 = 32\lambda C_0^2$.

Remark 2.1 (*[Derived]*, Paper 11, Remark 4.6 [11])**.** The 3 copies of irr_7 in the permutation character $\chi_{\mathrm{perm}} = \chi_1 \oplus \chi_3 \oplus \chi_3' \oplus 4\chi_6 \oplus 3\chi_7 \oplus 4\chi_8$ are governed by the 3×3 matrix M_7 of A restricted to the irr_7-isotypic component. Direct computation: $\det(\lambda I - M_7) = \lambda^3 - 12\lambda + 16 = (\lambda+4)(\lambda-2)^2$. The root $\lambda = -4$ accounts for Level 1 ($1 \cdot \mathrm{irr}_7$, tachyonic sector); the double root $\lambda = +2$ accounts for Level 4 ($2 \cdot \mathrm{irr}_7$, stable Fano sector). The same irrep irr_7 governs the Clifford structure (Paper 7), the mass spectrum (Paper 11), and the stability analysis of this paper — an algebraic coherence thread of the series.

2.3. The condensate and BF stability.

Theorem 2.2 (*[Derived]*, Paper 11 Thm 5.4 [11]). *The unique* $\mathrm{SO}(1,4)$*-invariant field configuration on* $\widetilde{\mathrm{dS}}_4$ *self-consistent with the Einstein equation* $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ *is* $\phi_k = C_0 = \mathrm{const}$*, where*

$$C_0^4 = \frac{3}{2048\pi G^3 M^2 \lambda}, \qquad r_s = 2GM, \tag{3}$$

with $T^{\mathrm{bulk}}_{\mu\nu}|_{C_0} = -g_{\mu\nu} \cdot 64\lambda C_0^4$*, giving* $\Lambda_{\mathrm{eff}} = 3/r_s^2$*.*

Proposition 2.3 (*[Derived]*, Paper 11 Prop. 5.7 [11]). *Under*

$$\lambda G M^2 < \tfrac{27\pi}{128} \approx 0.663 \qquad \textit{(BFcond)}, \tag{4}$$

the 7 Level-1 modes satisfy the de Sitter BF bound $m_1^2 > m_{\mathrm{BF}}^2 = -9/(4r_s^2)$ *and belong to the complementary series of* $\mathrm{SO}(1,4)$*: consistently quantisable, not physically unstable.*

The cavity bound $\lambda GM^2 < 2.14 \times 10^{-3}$ (Paper 14, Theorem 3.2 [14]) is $\approx 310\times$ stronger and is assumed throughout.

3. Step 1 — Classical Potential

3.1. Potential and EOM.

Proposition 3.1 (*[Derived]*). *The classical interaction potential on the* $\mathrm{SO}(1,4)$*-invariant background* $\phi_k = C$ *is*

$$V_{\mathrm{classical}}(C) = 64\lambda C^4. \tag{5}$$

Proof. From Theorem 2.2: $T^{\mathrm{bulk}}_{\mu\nu}|_C = -g_{\mu\nu} V(C)$ and $8\pi G \cdot 64\lambda C_0^4 = 3/r_s^2$ using (3). □

Proposition 3.2 (*[Derived]*, Paper 11 Prop. 5.2 [11]). *For* $\lambda > 0$*,* $\xi \geq 0$*: the scalar EOM at* $\phi_k = C_0$*,*

$$32\lambda C_0^3 + \tfrac{12\xi}{r_s^2} C_0 = 0, \tag{6}$$

has no positive solution. Hence $V'_{\mathrm{classical}}(C_0) = 256\lambda C_0^3 \neq 0$*:* C_0 *is* ***not*** *a stationary point of* $V_{\mathrm{classical}}$*.*

Remark 3.3 (*[Derived]*). The condensate C_0 is a gravitational background satisfying the Einstein equation (Theorem 2.2) but not the scalar EOM. The correct perturbative framework for this paper is the **background field method**: expand $\phi_k = C_0 + h_k$ (quantum fluctuations) and evaluate the Hessian at C_0. The physical stability question — are the BF-bounded modes quantum-mechanically well-defined? — has an affirmative answer under the cavity bound (Paper 14 [14]).

3.2. Saddle-point structure.

Theorem 3.4 (*[Derived]*). *The Hessian* $H_{kj} = 16\lambda C_0^2 (2\delta_{kj} + A_{kj})$ *at* C_0 *has:*

$$V''_{\mathrm{classical}}(C_0)\big|_{\mathrm{irr}_1} = +96\lambda C_0^2 > 0 \quad \textit{(condensate direction, stable)}, \tag{7}$$

$$V''_{\mathrm{classical}}(C_0)\big|_{\mathrm{irr}_7 \textit{ Level 1}} = -32\lambda C_0^2 < 0 \quad \textit{(7 tachyonic directions)}. \tag{8}$$

C_0 *is a* ***saddle point*** *of* $V_{\mathrm{classical}}$*: positive curvature in 63 directions (Levels 3–5), zero in 14 (Level 2,* $\xi = 0$*), negative in 7 (Level 1,* irr_7*);* $7 + 14 + 63 = 84$*.*

Corollary 3.5 (*[Derived]*). $\mathrm{Im}(W_{1\text{-}loop}) > 0$ *from growing QNM modes (Paper 18 [18]) is the dynamical manifestation of the Level-1 tachyons. The saddle-point structure and the metastability signal are mutually consistent.*

4. Step 2 — One-Loop Mass Correction

4.1. **Coleman–Weinberg formula.** The one-loop effective potential via the background field method (Coleman–Weinberg [20], de Sitter approximation valid at leading order in $R/m^2 \ll 1$ under the cavity bound):

$$V_{\text{1-loop}}(C) = \frac{1}{64\pi^2} \sum_{\alpha=1}^{84} m_\alpha^4(C) \left[\ln \frac{m_\alpha^2(C)}{\mu^2} - \frac{3}{2} \right], \tag{9}$$

where $m_\alpha^2(C) = 16\lambda C^2(2+\mu_\alpha)$ ($\xi = 0$). For Level 1 with $m_1^2 < 0$: analytic continuation $m^2 \to -|m^2| + i0^+$ gives $\ln m^2 = \ln|m^2| + i\pi$; the imaginary part reproduces $\mathrm{Im}(W) > 0$ (Paper 18, consistent); the real part alone enters $V_{\text{1-loop}}$.

4.2. **Direct correction to the Level-1 mass.**

Proposition 4.1 (*[Derived]*). *Let* e_1 *denote the Level-1 anti-uniform eigenvector of the ZD graph on a 12-vertex bipartite component* m *(adjacency eigenvalue* $\mu_1 = -4$*, bipartition* $A \cup B$*,* $|A| = |B| = 6$*; [11]). Its Hadamard (componentwise) square satisfies*

$$e_1^{\circ 2} = \tfrac{1}{\sqrt{12}}\, u_m, \tag{10}$$

where u_m *is the uniform Level-5 eigenvector of* m *(adjacency eigenvalue* $\mu_5 = +4$*). All off-diagonal one-loop couplings Level-1* $\leftrightarrow$ *Levels 1,2,3,4 vanish exactly by eigenspace orthogonality. The active one-loop coupling is* exclusively *Level-1* $\leftrightarrow$ *Level-5.*

Proof. The 12-vertex component m is 4-regular bipartite with bipartition $A \cup B$, $|A| = |B| = 6$ [11]. The anti-uniform eigenvector is $(e_1)_k = +1/\sqrt{12}$ for $k \in A$, $(e_1)_k = -1/\sqrt{12}$ for $k \in B$, $(e_1)_k = 0$ for $k \notin m$. The Hadamard square: $(e_1^{\circ 2})_k = (e_1)_k^2 = 1/12$ for all $k \in m$, independent of the bipartition sign. Since $(u_m)_k = 1/\sqrt{12}$ for $k \in m$, we have $(e_1^{\circ 2})_k = (1/\sqrt{12})(u_m)_k$, giving (10). The identity holds for every Level-1 mode: all seven irr_7 components have the same $\pm 1/\sqrt{12}$ bipartite structure [11].

For the off-diagonal couplings: the one-loop mixing matrix element is $e_1^T H'(0)\, w_\beta \propto (e_1^{\circ 2})^T w_\beta = (1/\sqrt{12})\, u_m^T w_\beta$. Since u_m has adjacency eigenvalue $\mu_5 = +4$, orthogonality of eigenspaces gives $u_m^T w_\beta = 0$ for $\beta \in$ Levels 1,2,3,4. The Level-1 $\leftrightarrow$ Level-5 channel is therefore the unique active off-diagonal coupling; all others vanish exactly. □

Theorem 4.2 (*[Derived]*). *The one-loop direct mass correction in the* irr_7 *direction at* $\mu^2 = 32\lambda C_0^2$ *is:*

$$\delta m_1^2\big|_{\text{direct}} \approx \frac{32\,\lambda^2 C_0^2}{3\pi^2} \approx +1.08\,\lambda^2 C_0^2 > 0. \tag{11}$$

The ratio to the classical mass:

$$\frac{|\delta m_1^2|_{\text{direct}}|}{|m_1^2|} \approx \frac{\lambda}{30} \ll 1 \quad \textit{for } \lambda \ll 1. \tag{12}$$

Proof. Since the interaction is quartic in the fields, $m'_\alpha(0) = 0$ for all mass eigenvalues (the first derivative w.r.t. the Level-1 fluctuation h vanishes by parity). The Coleman–Weinberg mass correction is therefore:

$$\delta m_1^2 = \frac{1}{32\pi^2} \sum_\alpha m_\alpha^2(0)\, (\hat{m}_\alpha^2)''(0)\, \mathrm{Re}\left[\ln \frac{|m_\alpha^2(0)|}{\mu^2} - 1 \right], \tag{13}$$

where $(\hat{m}_\alpha^2)''(0) \equiv \mathrm{d}^2 m_\alpha^2/\mathrm{d}h^2\big|_{h=0}$. By second-order perturbation theory:

$$(\hat{m}_1^2)''(0) = e_1^T H''(0)\, e_1 + 2 \sum_{\beta \neq 1} \frac{|e_1^T H'(0)\, w_\beta|^2}{m_1^2(0) - m_\beta^2(0)}. \tag{14}$$

Direct term. Computing $e_1^T H''(0) e_1$ from the 4-regular bipartite structure (using $(e_1^{\circ 2})^T A (e_1^{\circ 2}) = (1/12)\mu_5 = 1/3$):

$$e_1^T H''(0) e_1 = 16\lambda \cdot \tfrac{1}{3} + 32\lambda \cdot \tfrac{1}{3} = 16\lambda.$$

Level-5 Hellmann-Feynman term. By Proposition 4.1, the unique active channel gives $e_1^T H'(0) u_m = 64\lambda C_0/\sqrt{12}$:

$$\frac{2\,(64\lambda C_0/\sqrt{12})^2}{m_1^2 - m_5^2} = \frac{2 \times 4096\lambda^2 C_0^2/12}{-128\lambda C_0^2} = -\frac{16\lambda}{3}.$$

Sum: $(\hat{m}_1^2)''(0) = 16\lambda - 16\lambda/3 = 32\lambda/3$.
Closed-form correction (Level-1 term; at $\mu^2 = 32\lambda C_0^2$: $\mathrm{Re}[\ln(32\lambda C_0^2/32\lambda C_0^2) - 1] = -1$):

$$\delta m_1^2\big|_{\text{direct}} = \frac{(-32\lambda C_0^2)(32\lambda/3)(-1)}{32\pi^2} = \frac{32\lambda^2 C_0^2}{3\pi^2} \approx +1.08\,\lambda^2 C_0^2.$$

Scope. The contributions from Levels 3 and 4 require explicit eigenvectors of the 12-vertex bipartite component graph (open problem, see §7). Since $m_1^2 - m_3^2 = -64\lambda C_0^2 < 0$ and $m_1^2 - m_4^2 = -96\lambda C_0^2 < 0$, their Hellmann-Feynman contributions are negative, reducing the total direct correction below $+1.08\,\lambda^2 C_0^2$. The stated value is therefore an *upper bound* on the total direct correction. The conclusion of Corollary 4.3 is unaffected: $|-58.35\lambda^2 C_0^2| \gg |1.08\lambda^2 C_0^2|$ regardless of the sign of the remaining contributions. □

Corollary 4.3 (*[Derived]*)**.** *For the Level-1 sector to be stabilised by the direct one-loop correction alone, one would require $\lambda \gtrsim 30$ — deep in the non-perturbative regime. In the perturbative regime $\lambda \ll 1$, the direct correction is insufficient to flip the sign of m_1^2.*

5. Step 3 — Gravitational Backreaction under IIB-II

Under IIB-II (Paper 2, Conjecture 3.6 [2]): $N = \sum_k \phi_k^2$ is the algebraic correlate of gravity. Therefore the one-loop quantum correction to $\langle N \rangle$ is a correction to the gravitational field. The condensate equation $3/r_s^2 = 8\pi G\, V_{\text{eff}}(C_0)$ must be solved self-consistently at one loop.

5.1. Full one-loop effective potential.

Proposition 5.1 (*[Derived]*)**.** *At $\xi = 0$ and $\mu^2 = 32\lambda C_0^2$:*

$$V_{\text{1-loop}}(C_0) = \frac{\lambda^2 C_0^4}{64\pi^2} \sum_i \mathcal{N}_i c_i^2 \left[\ln\frac{|c_i|}{32} - \frac{3}{2}\right] \approx -233.4\,\lambda^2 C_0^4, \tag{15}$$

where $c_i = 16(2 + \mu_i)$.

Level	$\mathcal{N}_i$	c_i^2	$\ln(\lvert c_i\rvert/32) - 3/2$	$\mathcal{N}_i c_i^2 \times [\cdots]$
5	*7*	*9216*	$\ln 3 - 3/2 \approx -0.401$	$-25\,869$
4	*14*	*4096*	$\ln 2 - 3/2 \approx -0.807$	$-46\,267$
3	*42*	*1024*	$-3/2$	$-64\,512$
2	*14*	*0*	—	0
1	*7*	*1024*	$-3/2$	$-10\,752$
Σ				$-147\,400$

$V_{\text{1-loop}}(C_0) = 0$ *at* $\mu^2 = 13.58\,\lambda C_0^2$*; for all* $\mu^2 > 13.58\,\lambda C_0^2$*:* $V_{\text{1-loop}} < 0$.

5.2. Self-consistent condensate.

Theorem 5.2 (*[Derived]*)**.** *The self-consistent condensate $C_0^{(1)} = C_0(1+\delta)$ satisfying $3/r_s^2 = 8\pi G\, V_{\rm eff}(C_0^{(1)})$ has:*

$$\delta = \frac{-V_{1\text{-}loop}(C_0)}{4\,V_{\rm classical}(C_0) + V'_{1\text{-}loop}(C_0)\cdot C_0} \approx \frac{233.4\,\lambda^2 C_0^4}{256\,\lambda C_0^4} = 0.912\,\lambda + \mathcal{O}(\lambda^2). \tag{16}$$

In particular $C_0^{(1)} > C_0$: the quantum-corrected condensate is larger than its classical value.

Proof. Expand $V_{\rm eff}(C_0+C_0\delta)$ to first order in δ and solve for δ imposing the self-consistent Einstein equation. The denominator is $256\lambda C_0^4 - 933.6\lambda^2 C_0^4 \approx 256\lambda C_0^4$ for $\lambda \ll 1$. □

The physical mechanism: $V_{\text{1-loop}}(C_0) < 0$ means quantum fluctuations deepen the effective potential. To maintain the gravitational constraint, the condensate must increase. This is the gravitational backreaction of the quantum field N on the geometry under IIB-II.

5.3. Backreaction to the Level-1 mass.

Theorem 5.3 (*[Derived]*)**.** *The shift $C_0 \to C_0^{(1)}$ modifies the Level-1 classical mass as:*

$$\delta m_1^2\big|_{\rm back} = -64\lambda C_0^2\cdot\delta = -64\lambda C_0^2 \times 0.912\lambda \approx -58.35\,\lambda^2 C_0^2. \tag{17}$$

This is negative (destabilising) and dominates the direct correction by a factor of ≈ 54.

The ratio of gravitational backreaction to direct correction is:

$$\frac{|\delta m_1^2|_{\rm back}|}{|\delta m_1^2|_{\rm direct}|} = \frac{|V_{\text{1-loop}}(C_0)|/(4C_0^2)}{32\lambda^2 C_0^2/(3\pi^2)} = \frac{233.4\,\lambda^2 C_0^4/(4C_0^2)}{32\lambda^2 C_0^2/(3\pi^2)} = \frac{58.35}{1.08} \approx 54, \tag{18}$$

where the numerator uses $|V_{\text{1-loop}}(C_0)| = 147\,426\,\lambda^2 C_0^4/(64\pi^2) \approx 233.4\,\lambda^2 C_0^4$ (Proposition 5.1, log-factors evaluated at $\mu^2 = 32\lambda C_0^2$), and the factor $1/4$ arises from $\delta m_1^2|_{\rm back} = (\partial m_1^2/\partial C_0)\cdot\delta C_0 = -64\lambda C_0\cdot(-V_{\text{1-loop}})/(256\lambda C_0^3) = V_{\text{1-loop}}/(4C_0^2)$.

The physical origin of the asymmetry: the backreaction involves $V_{\text{1-loop}}$ summed *coherently over all 84 modes* — after the logarithmic weights and the $1/(64\pi^2)$ prefactor, the effective coefficient is 233.4. The direct correction involves only the *Level-1 self-energy* in one component m, with effective coefficient $32/(3\pi^2) \approx 1.08$. Under IIB-II, this asymmetry has a transparent physical meaning: the backreaction is a **collective gravitational response** of the full scalar sector (all of $N = \sum_k \phi_k^2$ is shifted), whereas the direct correction is a **local quantum effect** in the irr_7 sector alone.

5.4. Main result.

Theorem 5.4 (*[Derived]*, at $\mu^2 = 32\lambda C_0^2$)**.**

$$\boxed{m_1^{2,\rm eff} = -32\lambda C_0^2\big(1 + 1.79\,\lambda + \mathcal{O}(\lambda^2)\big) < 0.} \tag{19}$$

Corollary 5.5 (*[Derived]*, $\mu^2 > 13.6\,\lambda C_0^2$; *[Conjecture]*, μ-independent)**.** ***Case (a): metastability is enhanced at one loop. IIB is perturbatively consistent.*** *The Level-1 tachyons become more tachyonic by 1.79λ fractionally at one loop. This is self-consistent with* $\mathrm{Im}(W) > 0$ *(Paper 18) and with* $\tau_{\rm grow} \approx 1.36\, r_s/c$ *(Papers 15–17* [15, 16, 17]*).*

Under IIB-II: the one-loop gravitational field amplifies the tachyonic sector. Gravity does not self-cure the instability; it deepens it. This is a falsifiable consequence of IIB-II: any alternative theory where N plays a different role would predict a different sign for $\delta m_1^2|_{\rm back}$.

Remark 5.6 (*[Derived]*)**.** The coefficient 1.79λ depends on $\mu^2 = 32\lambda C_0^2$. Case (a) is valid for all $\mu^2 > 13.6\,\lambda C_0^2$ (all physically motivated scales). A μ-independent derivation is OP 20.1.

5.5. First IIB-II loop prediction and $\langle N\rangle$.

Proposition 5.7 (*[Derived]*)**.** *The one-loop correction to the gravitational correlate (standard tadpole, dim. reg. MS-bar):*

$$\mathrm{Re}\langle N\rangle_{1\text{-}loop} = \frac{1}{16\pi^2}\sum_i \mathcal{N}_i\, m_i^2\,\mathrm{Re}\left[\ln\frac{m_i^2}{\mu^2} - 1\right]. \tag{20}$$

The exact sum identity $\sum_i \mathcal{N}_i(2+\mu_i) = 168$ *(*[Derived]*, traceless for the ZD graph spectrum). At* $\mu^2 = 32\lambda C_0^2$*:*

$$\boxed{\mathrm{Re}\langle N\rangle_{1\text{-}loop} \approx -8.41\,\lambda C_0^2 < 0.} \tag{21}$$

(Level contributions in units λC_0^2*:* $+66.5 - 274.9 - 1344.0 + 0 + 224.0 = -1328.4$*; then* $-1328.4/(16\pi^2) \approx -8.41$*.)*

Theorem 5.8 (*[Conjecture]*, IIB-II)**.** *Under Conjecture 3.6 of Paper 2* [2] *(N is the algebraic correlate of gravity):*

$$\frac{\langle N\rangle_{1\text{-}loop}}{C_0^2} \approx -8.41\,\lambda < 0 \qquad \text{at } \mu^2 = 32\lambda C_0^2. \tag{22}$$

Quantum fluctuations of the S_{84} *fields reduce the expectation value of the gravitational correlate* N *below its classical value. This constitutes a negative,* $\mathcal{O}(\lambda)$ *quantum gravitational correction — the first quantitative one-loop prediction of the norm-gravity conjecture, conditional on IIB-II.*

Proposition 5.9 (*[Derived]*)**.** *The Analytical Mean Theorem* $\mathbb{E}_v[N(av)] = N(a)$ *(Papers 1–2, unconditional* [1, 2]*) is not violated by* $\langle N\rangle_{1\text{-}loop} \neq 0$*.*

Proof. The AMT is a classical algebraic identity holding pointwise for every field configuration ϕ: it constrains the algebraic structure of N at each point in field space, not the quantum path-integral average over configurations. The one-loop correction is a quantum vacuum expectation value, a distinct mathematical object. No tension. □

5.6. Quantum GHY boundary correction.

Proposition 5.10 (*[Derived]*, order of magnitude)**.** *The one-loop quantum GHY correction, arising from the boundary heat kernel at* $r = r_s$*, is suppressed by* $e^{-m_k r_s} \sim e^{-\sqrt{\lambda G M^2}} < e^{-0.046}$ *under the cavity bound. This amounts to* $\mathcal{O}(\sqrt{\lambda G M^2}) < 5\%$ *of the bulk one-loop result and is sub-leading. Level-1 modes satisfy the BF bound throughout the interior:* $f_{\mathrm{in}}(r) \to 0$ *at the shell sets the effective boundary mass to zero, so no additional boundary instability arises.*

6. Discussion

6.1. Structural coherence diagram.

Quantity	Status
$V''(C_0)\vert_{\mathrm{irr}_1} = +96\lambda C_0^2 > 0$	*[Derived]*, stable
$V''(C_0)\vert_{\mathrm{irr}_7} = -32\lambda C_0^2 < 0$	*[Derived]*, saddle
$\delta m_1^2\vert_{\mathrm{direct}} \approx +1.08\lambda^2 C_0^2$	*[Derived]*, $\vert\cdot\vert/\vert m_1^2\vert \approx \lambda/30$
$\delta m_1^2\vert_{\mathrm{back}} \approx -58.4\lambda^2 C_0^2$	*[Derived]*, dominates by $\times 54$
$m_1^{2,\mathrm{eff}} = -32\lambda C_0^2(1+1.79\lambda)$	*[Derived]*, case (a)
$\langle N\rangle_{1\text{-loop}}/C_0^2 \approx -8.41\lambda$	*[Conjecture]*, IIB-II
$\mathrm{Im}(W) > 0 \leftrightarrow m_1^{2,\mathrm{eff}} < 0$	*[Derived]*, mutually consistent
BF-stable under cavity bound	*[Derived]*

C_0 is not a true vacuum in the traditional sense. It is a BF-stable, metastable, gravitationally self-consistent background:

(i) *BF-stable*: under the cavity bound, all 84 modes satisfy $m_i^2 > m_{\mathrm{BF}}^2$. The Level-1 tachyons are in the complementary series of $\mathrm{SO}(1,4)$ — not physically unstable in de Sitter.

(ii) *Metastable*: $\mathrm{Im}(W) > 0$ and $m_1^{2,\mathrm{eff}} < 0$ confirm metastability. The condensate decays on timescale $\tau_{\mathrm{grow}} \approx 1.36\, r_s/c$.

(iii) *Gravitationally self-consistent*: the one-loop corrected condensate $C_0^{(1)} = C_0(1 + 0.912\lambda)$ satisfies $3/r_s^2 = 8\pi G\, V_{\mathrm{eff}}(C_0^{(1)})$. It is real and positive.

IIB does not require reformulation. The metastability is a prediction, not a defect.

6.2. **The condensate is not a local minimum — and that is correct.** This paper establishes the conceptually sharper statement: C_0 is a gravitational condensate for which the notion of "scalar potential minimum" is replaced by "solution of the coupled Einstein + background field equations". The physical stability question — are the BF-bounded modes quantum-mechanically well-defined? — has an affirmative answer under the cavity bound.

6.3. **The dominant role of backreaction under IIB-II.** The factor-54 dominance of the backreaction over the direct correction is a distinctive feature of the IIB-II framework. In a theory where N is merely a scalar observable (metric fixed, no backreaction), only the direct correction $\delta m_1^2|_{\mathrm{direct}} \approx +1.08\lambda^2 C_0^2$ would appear, partially stabilising the tachyon. Under IIB-II, the gravitational backreaction reverses this: the quantum correction to $\langle N\rangle$ increases C_0, deepening the tachyonic well.

This is qualitatively different physics. The two frameworks — (i) S_{84} on fixed de Sitter and (ii) S_{84} with self-consistent IIB-II backreaction — predict different signs for the effective tachyon mass correction. This difference is in principle observable.

7. Open Problems

OP 20.1 (μ-independent mass correction). *[Open]*Compute the two-loop renormalisation group equation for the effective Level-1 mass and confirm case (a) holds at all renormalisation scales. This would promote Corollary 5.5 from *[Derived]*, $\mu^2 = 32\lambda C_0^2$ to *[Derived]*, μ-independent.

OP 20.2 (Quantum GHY via Black Circle propagator). *[Open]*Construct the bulk-to-boundary propagator for the Black Circle geometry (two-domain, Eddington–Finkelstein frame) and compute the exact one-loop quantum GHY correction to $V_{\text{1-loop}}$.

OP 20.3 ($\delta G_{\mathrm{eff}}/G$ from IIB-II). *[Conjecture]*, *[Open]*Derive the explicit formula relating $\langle N\rangle_{\text{1-loop}}$ to the renormalisation of G from IIB-II first principles. The result of Theorem 5.8 provides the input; the IIB-II mapping $N \to G$ at the quantum level requires a precise operator statement.

OP 20.4 (β-function and two-loop RGE — Paper 21). *[Derived]* (leading order), *[Open]* (two-loop)

The one-loop β-function of S_{84} on flat spacetime:

$$\beta(\lambda) = \frac{d\lambda}{d\ln\mu} = \frac{3}{16\pi^2}\lambda^2\,\mathrm{Tr}(T^2), \tag{23}$$

where T_{ab} is the coupling tensor from the ZD graph. Using $\mathrm{Tr}(A^2) = 84 \times 4 = 336$ (4-regularity):

$$\beta(\lambda) \approx \frac{63\lambda^2}{\pi^2} > 0. \tag{24}$$

The positive β-function places the Landau pole at $\mu_L = \mu_0 \exp(\pi^2/(63\lambda))$ — exponentially large for $\lambda \ll 1$, well above all physical scales. The two-loop RGE then yields the μ-independent derivation of the case (a) conclusion, closing OP 20.1. Proposed as the first section of Paper 21.

8. Conclusions

8.1. **What this paper has done. Step 1** established $V_{\text{classical}}(C) = 64\lambda C^4$, that C_0 is a gravitational background (not a scalar minimum), and that the Hessian at C_0 has 7 negative eigenvalues $m_1^2 = -32\lambda C_0^2$ (irr_7 sector) — a classical saddle point. *[Derived]*

Step 2 computed the CW one-loop correction via the background field method. The structural result $e_1^{\circ 2} = (1/\sqrt{12})u_m$ confines the leading coupling to Level-1 $\leftrightarrow$ Level-5, giving $\delta m_1^2|_{\text{direct}} \approx +1.08\lambda^2 C_0^2$, insufficient to stabilise the tachyon at weak coupling. *[Derived]*

Step 3 closed the IIB-II backreaction gap. Under the self-consistent Einstein equation at one loop, $C_0^{(1)} = C_0(1 + 0.912\lambda)$. The gravitational backreaction dominates the direct correction by a factor of 54, giving $m_1^{2,\text{eff}} = -32\lambda C_0^2(1 + 1.79\lambda) < 0$: case (a) confirmed. *[Derived]*, scheme-dep.; *[Conjecture]*, scheme-indep.

First IIB-II loop prediction: $\langle N\rangle_{\text{1-loop}}/C_0^2 \approx -8.41\lambda$. *[Conjecture]*, IIB-II

AMT compatibility confirmed. Quantum GHY sub-leading under the cavity bound. *[Derived]*

8.2. **What this paper has not done.**

- Provided a μ-independent derivation of case (a) (OP 20.1).
- Computed the exact quantum GHY contribution (OP 20.2).
- Derived the explicit $\delta G_{\text{eff}}/G$ formula (OP 20.3).
- Computed $m''_\alpha(0)$ for Levels 2,3,4 within the ZD graph components analytically (requires explicit eigenvectors of the 12-vertex 4-regular bipartite component graph).
- Addressed the β-function (OP 20.4, proposed for Paper 21: $\beta(\lambda) = 63\lambda^2/\pi^2 > 0$, Landau pole at exponentially large scale).

8.3. **The sentence.** The condensate C_0 of S_{84} is a BF-stable, metastable, gravitationally self-consistent background whose tachyonic sector is **enhanced** (not cured) by one-loop quantum corrections; the gravitational backreaction under IIB-II dominates the direct eigenvalue correction by a factor of ≈ 54 and confirms metastability as a robust feature of the norm-gravity framework.

Acknowledgements

Structural analysis, Coleman–Weinberg computation, gravitational backreaction derivation, and first IIB-II loop prediction The author thanks Claude (Anthropic) for assistance with algebraic computations and LaTeX preparation. Numerical computations on AMD Threadripper 2950X (Python 3, NumPy, SciPy, mpmath at 50 d.p.). The author takes full responsibility for any errors.

References

[1] G. Levratti, *Sign-Parity Classification of Zero-Divisors in Cayley–Dickson Algebras*, v2, Zenodo (2026), doi:10.5281/zenodo.20315751.

[2] G. Levratti, *The Informational Identity Boundary: Algebraic Foundations and Physical Conjectures*, v4, Zenodo (2026), doi:10.5281/zenodo.20411845.

[3] G. Levratti, *Sign Structure, Automorphism Groups and the ZD Zeta Function of Cayley–Dickson Algebras*, v4, Zenodo (2026), doi:10.5281/zenodo.20397587.

[4] G. Levratti, *Zero-Divisors in Cayley–Dickson Algebras: Fano Geometry, Automorphisms and the Weil Zeta Function*, v10, Zenodo (2026), doi:10.5281/zenodo.20417859.

[5] G. Levratti, *Active Stabilizers in Reed–Muller CSS Codes from Cayley–Dickson Zero-Divisors*, v1, Zenodo (2026), doi:10.5281/zenodo.20431754.

[6] G. Levratti, *The Factorization Threshold in Cayley–Dickson Integer Algebras*, v1, Zenodo (2026), doi:10.5281/zenodo.20442390.

[7] G. Levratti, *A Clifford Representation of the Fano Zero-Divisors*, v1, Zenodo (2026), doi:10.5281/zenodo.20445208.

[8] G. Levratti, *Fano Duality, Projective Orthogonality and the CSS Bridge*, v1, Zenodo (2026), doi:10.5281/zenodo.20445734.

[9] G. Levratti, *A Quartic Contact Interaction with Fano–CSS Structure*, v1, Zenodo (2026), doi:10.5281/zenodo.20506165.
[10] G. Levratti, *Loop, Threshold, and Residue: The Algebraic Structure of the $\mathcal{A}_3 \leftrightarrow \mathcal{A}_4$ Transition*, v1, Zenodo (2026), doi:10.5281/zenodo.20507204.
[11] G. Levratti, *Canonical Quantisation of the Black Circle Interior*, v1, Zenodo (2026), doi:10.5281/zenodo.20508375.
[12] G. Levratti, *Boundary Conditions and Spinorial Statistics of the Black Circle Interior*, v1, Zenodo (2026), doi:10.5281/zenodo.20509490.
[13] G. Levratti, *Quasi-Normal Modes of the Black Circle Interior*, v1, Zenodo (2026), doi:10.5281/zenodo.20509979.
[14] G. Levratti, *Black Circle Hypergeometric Reduction and Cavity Bound*, v1, Zenodo (2026), doi:10.5281/zenodo.20553409.
[15] G. Levratti, *The Kummer Connection Matrix for the Black Circle*, v1, Zenodo (2026), doi:10.5281/zenodo.20554392.
[16] G. Levratti, *The Limit-Circle Obstacle for Black Circle Quasi-Normal Modes*, v1, Zenodo (2026), doi:10.5281/zenodo.20554795.
[17] G. Levratti, *The Physical Boundary Condition at $r = 0$ for the Black Circle*, v1, Zenodo (2026), doi:10.5281/zenodo.20555042.
[18] G. Levratti, *Vacuum Instability and Majorana Spinors of the Black Circle*, v2, Zenodo (2026), doi:10.5281/zenodo.20562197.
[19] G. Levratti, *The Majorana Sector of the Black Circle*, v1, Zenodo (2026), doi:10.5281/zenodo.20571085.
[20] S. R. Coleman and E. J. Weinberg, Radiative corrections as the origin of spontaneous symmetry breaking, *Phys. Rev. D* **7** (1973), 1888–1910.

Independent Researcher, Modena, Italy
Email address: giovanni.levratti.research@gmail.com