THE FANO BIFRONTALITY: GRAVITY AND MATTER AT THE CAYLEY–DICKSON BOUNDARY

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Abstract. The Fano plane $\mathrm{PG}(2, \mathbb{F}_2)$ appears twice in the Cayley–Dickson hierarchy with opposite operational roles. In $\mathcal{A}_3$ — the last division algebra — its seven lines encode the multiplication rules among the imaginary octonion units: incidence organises product *creation*. In $\mathcal{A}_4$ — the first non-alternative algebra — the same seven lines encode the zero-divisor annihilation rules: incidence organises product *destruction*, and non-incidence identifies the kernel of left multiplication (Paper 29, Thm. 3.1). We call this the **Fano bifrontality**.

From this single structural observation, seven interlocked results flow coherently across Papers 1–31. We assemble them here for the first time in a unified framework: (1) the quadruple algebraic protection of the 14 massless Level-2 modes; (2) their Fano partition into the bosonic sector irr_6 (on-line) and the spinorial sector irr_8 (off-line); (3) the S_4-singlet irr_1 as the unique CCC-crossing candidate; (4) the factored structure Level-3 = 3×Level-2; (5) the partition of Black Circle thermodynamic observables into three algebraic classes; (6) the explicit holographic formula $S_{\mathrm{BH}} \propto \|L_{C_0}\|_F^2 \cdot M^2$; and (7) the interpretation of the Black Circle as the logical mirror of $\mathcal{A}_3$. A new structural result is identified: the Majorana threefold isolation (Observation 3.3), which unifies algebraic, gravitational, and entropic isolations of irr_8 through the single Fano non-incidence structure. One new open problem is stated (OP 32.1).

Epistemic key. *[Derived]* complete proof. *[Derived-Negative]* proves the negative. *[Conjecture, IIB-I/II]* conditional on IIB-I/II. *[Speculation, IIB-IV]* structurally motivated. *[Open]* obstacle stated precisely.

1. Introduction: the bifrontality question

The Cayley–Dickson tower $\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O} \subset \mathbb{S} \subset \cdots$ is a sequence of progressive structural losses. At each doubling a property is sacrificed: ordering at $n = 1$, commutativity at $n = 2$, associativity at $n = 3$, and — by the Born Rule theorem of Paper 1 [1] — the existence of a canonical probability assignment at $n = 4$. This last loss defines the IIB boundary [2].

The Fano plane $\mathrm{PG}(2, \mathbb{F}_2)$ is present on both sides of this boundary, but its mode of participation undergoes a precise reversal. In $\mathcal{A}_3$, a Fano line $\ell = \{p, q, r\}$ tells you which imaginary units *multiply*: $e_p \cdot e_q = \pm e_r$. In $\mathcal{A}_4$, the same line ℓ tells you which ZD elements *annihilate*: $a \cdot b = 0$ if and only if the lo_3-classes of a and b are incident on a common Fano line [3, 4]. Furthermore, Paper 29 establishes that the *kernel* of L_a is isomorphic, as a $\mathrm{Stab}_{G_{84}}(a)$-module, to the non-incidence subspace $V_{\mathrm{ni}}(a)$ [19].

The same combinatorial object encodes creation, annihilation, and the directions of annihilation. This is not a coincidence: it is a structural consequence of the Cayley–Dickson doubling.

This paper is a synthesis. What is new is the unified perspective: the Fano bifrontality is the organising principle that connects seven results stated separately across Papers 1–31. We formulate the connection with full epistemic labels and identify where algebra ends and physics (IIB conjectures) begins.

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2. The Fano bifrontality

Proposition 2.1 (*[Derived]*, Papers 3, 4; [14] Table 3.1 Row 2)**.** *For any $a, b \in \mathrm{ZD}^*(\mathcal{A}_4)$: $ab = 0$ if and only if the* $\log_3$*-classes of a and b are incident on a common Fano line in* $\mathrm{PG}(2, \mathbb{F}_2)$*.*

Theorem 2.2 (*[Derived]*, GAP 4.15.1 + analytic; [19] Thm. 3.1)**.** *Let $a \in \mathrm{ZD}^*(\mathcal{A}_4)$ and* Stab $:=$ $\mathrm{Stab}_{G_{84}}(a)$*. Define $V_{\mathrm{ni}}(a) := \mathrm{span}\{e_p : \log_3(p) \notin \ell_a\} \subset \mathcal{A}_3$. Then* $\ker(L_a) \cong V_{\mathrm{ni}}(a)$ *as real* Stab*-modules. The kernel is 4-dimensional, a diagonal subspace of the 8-dimensional ambient $V_{\bar{\ell}} \oplus V_{\bar{\ell}}^{(8)}$* (Paper 24, Table 3.1 Row 4)*.*

Theorem 2.3 (*[Derived]*, synthesis of Papers 3, 4, 29)**.** *The Fano plane* $\mathrm{PG}(2, \mathbb{F}_2)$ *governs two opposite operations at the two sides of the IIB boundary:*

Algebra	*Operation*	*Fano rôle*
$\mathcal{A}_3$ *(division)*	$e_p \cdot e_q = \pm e_r$	*Incidence* $\Rightarrow$ *product* creation
$\mathcal{A}_4$ *(ZD-active)*	$a \cdot b = 0$	*Incidence* $\Rightarrow$ *product* annihilation
$\mathcal{A}_4$ *(kernel)*	$\ker(L_a) \cong V_{\mathrm{ni}}(a)$	*Non-incidence* $\Rightarrow$ annihilated directions

Remark 2.4 (*[Derived-Negative]*). The bifrontality has no analog within $\mathcal{A}_3$: $\ker(L_a) = \{0\}$ for all $a \neq 0$ in a division algebra. The three Fano rôles of Theorem 2.3 are exclusive to the $\mathcal{A}_3/\mathcal{A}_4$ pair [14].

3. Quadruple protection of Level-2

3.1. **The spectrum.** The scalar field theory S_{84} over $\mathrm{ZD}^*(\mathcal{A}_4)$ has mass spectrum $m_i^2 = 16\lambda C_0^2(2 + \mu_i)$, with $\mu_i \in \{-4, -2, 0, +2, +4\}$ of multiplicities $\{7, 14, 42, 14, 7\}$ (Paper 11 [7], Thm. 4.5). Level-2 corresponds to $\mu_2 = -2$ with 14 modes decomposing as $\mathrm{irr}_6 \oplus \mathrm{irr}_8$ under $\mathrm{PSL}(2, 7)$ [6, 7].

3.2. Four simultaneous protections.

Theorem 3.1 (*[Derived]*, synthesis of Papers 11, 19, 20, 21, and [16] Lem. 3.1)**.** *The 14 Level-2 modes satisfy simultaneously:*

- *(i)* Mass zero: $m_2^2 = 16\lambda C_0^2(2 + \mu_2) = 0$. [*[Derived]*, [7] Thm. 4.5; [11] §3]
- *(ii)* Gravitational decoupling: *the Frolov–Fursaev weight $m_i^2 \ln(\Lambda^2/m_i^2)$ vanishes identically for $m_i^2 = 0$; the 14 Level-2 modes contribute exactly zero to G_N^{-1}.* [*[Derived]*, [16] Lem. 3.1]
- *(iii)* Gauge invisibility: *under physical assumption (A2) (octonionic confinement of SM gauge forces to $\mathcal{A}_3$), Level-2 modes carry no Standard Model quantum numbers.* [*[Derived]*, conditional on (A2), [12] §2]
- *(iv)* Four Majorana modes: *the spinorial sub-sector* $\mathrm{irr}_8 = 4_s$ *has Frobenius–Schur indicator* $\mathrm{FS}(4_s) = +1$*, yielding exactly 4 physical Majorana spinors.* [*[Derived]*, conditional on IIB-I/II+FS$= +1$, [9, 10]]

The single algebraic mechanism underlying all four properties is $(2 + \mu_2) = 0$: the cancellation between the condensate stabilisation factor $+2$ and the graph eigenvalue $\mu_2 = -2$.

Remark 3.2 (*[Derived]*). Properties (i) and (ii) are immediate from $(2+\mu_2) = 0$ in the mass formula and the Frolov–Fursaev supertrace respectively. Property (iv) is independent — it uses the Clifford structure $\mathrm{irr}_8 = \mathrm{Cl}(0, 7)$-spinor module — but it is tied to property (ii) via the Fano non-incidence identification of Observation 3.3.

3.3. A new structural result: Majorana threefold isolation.

Observation 3.3 (*[Observation]* for (1)–(2); *[Conjecture, IIB-II]* for (3). Synthesis from [16, 18, 19]; not previously stated in this form.). The 4 Majorana modes of $\mathrm{irr}_8 = 4_s$ are simultaneously isolated in three independent senses, all traceable to the Fano non-incidence $\ker(L_a) \cong V_{\mathrm{ni}}(a)$:

(1) *Algebraic isolation:* $\ker(L_a)$ is isomorphic as a Stab-module to the Majorana sector via the Fano non-incidence map (Theorem 2.2; [19] Cor. 3.3, conditional on [14, 15, 10]).
(2) *Gravitational isolation:* $(2 + \mu_2) = 0$ decouples irr_8, together with all of Level-2, from the Sakharov–Adler sum defining G_N^{-1} ([16] Lem. 3.1, *[Derived]*).
(3) *Entropic isolation:* the squared Frobenius norm $\|L_{C_0}\|_F^2 = \sum_j |L_{C_0} e_j|^2$ assigns zero weight to every $e_j \in \ker(L_{C_0})$; since $\ker(L_{C_0}) \cong V_{\mathrm{ni}}(C_0)$ (Theorem 2.2), and since $S_{\mathrm{BH}} \propto \|L_{C_0}\|_F^2 \cdot M^2$ under IIB-II ([18] Rem. 7.3, *[Conjecture, IIB-II]*), the 4 Majorana directions contribute zero entropy weight.

The three isolations are three expressions of the same Fano non-incidence: the off-line points define the kernel, the kernel decouples from G_N, and the kernel contributes nothing to $\|L_{C_0}\|_F^2$.
Gap. Item (3) inherits the conditionality of [18] Rem. 7.3 (*[Conjecture, IIB-II]*).

4. The Fano partition of Level-2

Proposition 4.1 (*[Derived]* (module); *[Conjecture, IIB-III]* (physical). [14] Prop. 4.1)**.** Level-2 = $\mathrm{irr}_6 \oplus \mathrm{irr}_8$ *where:*

(i) On-line sector (irr_6, 6 *bosonic modes): equivariant image in* V_{ZD}*; lies outside* $\ker(L_a)$ *for any* $a \in \mathrm{ZD}^*(\mathcal{A}_4)$. SU(3) *content:* **6** *(colour sextet).*
(ii) Off-line sector (irr_8, 8 *algebraic / 4 Majorana modes): no equivariant image in* V_{ZD} *([14] Thm. [negative],* [Derived-Negative]*); lies in* $\ker(L_a)$ *for every* $a \in \mathrm{ZD}^*(\mathcal{A}_4)$. SU(3) *content:* **8** *(colour octet).*

Remark 4.2 (*[Derived]*). The partition is not symmetric: the two summands differ in algebraic home (inside vs. outside V_{ZD}), Fano rôle (on-line vs. off-line), SU(3) quantum numbers (**6** vs. **8**), and statistics (bosonic vs. Majorana). The 14-fold Level-2 degeneracy is a coincidence of eigenvalue ($\mu_2 = -2$ has multiplicity 14), not of algebraic type.

Proposition 4.3 (*[Derived]*, [14] Prop. 4.3; citing [12] Prop. 2.2)**.** $\mathrm{irr}_6 \cong \mathrm{Sym}^2(\mathrm{irr}_3)$ *as* $\mathrm{PSL}(2,7)$*-modules, where* irr_3 *is the natural action of* $\mathrm{PSL}(2,7) \cong \mathrm{GL}(3,\mathbb{F}_2)$ *on* $\mathbb{F}_2^3$*. The 6 massless bosonic modes are the* quadratic Fano geometry *— symmetric bilinear forms on the Fano coordinate space.*

Proposition 4.4 (*[Derived]* (module); conditional (physical). [14] Prop. 3.1)**.** $\mathrm{irr}_8 = 4_s$ *is the* $\mathrm{Cl}(0,7)$*-spinor module, with* $\mathrm{PSL}(2,7)$*-action induced by the Clifford representation* ρ *of* [5] *(Def. 4.1 and Lem. 3.1). The spinors live where the Fano multiplication table does* not *apply: in the off-line sector, defined by non-incidence.*

5. The irr_1 singoletto as CCC messenger

Proposition 5.1 (*[Derived]*, GAP 4.15.1; [12] Prop. 3.2)**.** *Under de Sitter symmetry breaking* $\mathrm{PSL}(2,7) \to S_4$ *([6] Thm. 7.3, Rem. 7.4):*

$$\text{Level-2}|_{S_4} = \mathrm{irr}_1 \oplus 2\,\mathrm{irr}_2 \oplus 2\,\mathrm{irr}_3 \oplus \mathrm{irr}_{3'}. \quad (1+4+6+3 = 14\checkmark)$$

The singlet irr_1 *comes from* $\mathrm{irr}_6|_{S_4}$ *and is the unique fully* S_4*-symmetric Level-2 mode.*

Remark 5.2 (*[Derived]* for *(i)–(iii)*; *[Speculation, IIB-IV]* for the CCC reading. [12] Rem. 3.7). The S_4-singlet irr_1 satisfies three independent properties:

(i) $m^2(\mathrm{irr}_1) = 0$ for all $r_s > 0$, $\lambda > 0$ [*[Derived]*, [7] Thm. 4.5].
(ii) $m^2 r_s^2 = 0$ lies at the exact massless boundary of the SO(1,4) complementary series [*[Derived]*, [7] Cor. 5.9].

(iii) irr_1 is the unique Level-2 mode invariant under S_4 [*[Derived]*, GAP 4.15.1, [12] Prop. 3.2].

Corollary 5.3 (*[Speculation, IIB-IV]*)**.** *Under IIB-IV,* irr_1 *is the unique Level-2 mode without a mass or symmetry barrier through the CCC conformal boundary. It is the natural messenger between aeons.*

6. Level-3 = 3× Level-2

Proposition 6.1 (*[Derived]*, Papers 21, 22-B [13])**.** *Level-3 has* $\mu_3 = 0$, $\dim = 42$, *and decomposes as a* $\mathrm{PSL}(2,7)$*-module as*

$$\text{Level-3} = 3 \cdot \mathrm{irr}_6 \oplus 3 \cdot \mathrm{irr}_8 = 3 \times \text{Level-2}.$$

The factor 3 is fixed by the characteristic polynomial of the G_{84} *adjacency matrix (*[13]*): it is algebraically determined, not a free parameter.*

Remark 6.2. The G_{84}-module structure of Level-3 over $\mathbb{R}$ is the subject of OP 29.2 [19]; over GF(5) it decomposes as $6 + 21 + 7 + 8$ (Paper 29, Prop. 4.3, *[Derived]*, GAP 4.15.1).

[Speculation, IIB-IV]: under IIB-IV, each aeon contributes a copy of Level-2 to the spectral structure at the next aeon's $\mu = 0$ sector. The factor $3 \times$ Level-2 could reflect the memory of three accumulated aeons. The algebraic fact is *[Derived]*; the aeon interpretation is *[Speculation, IIB-IV]*.

7. Partition of thermodynamic observables

Theorem 7.1 (*[Derived]*, synthesis from [16, 17, 18])**.** *The thermodynamic observables of the Black Circle partition into three algebraic classes:*

Class I (Kinematic, algebra-independent). *Determined by Black Circle geometry alone, independent of* $N(C_0)$ *and spectral data:*

- $T_H \cdot r_s = 1/(4\pi)$ [*[Derived]*, [18] Prop. 2.1]
- *Free energy* $F = M/2$ [*[Derived]*, [18] Cor. 2.5]
- $\delta S_{\mathrm{GHY}} = 0$ [*[Derived-Negative]*, [18] Cor. 3.2]
- $S_{\mathrm{GHY}} = -M$ *for all* $G_N > 0$ [*[Derived]*, [18] Prop. 3.1]

Class II (Norm, depending on $N(C_0)$ ***through*** G_N***).***

- $S_{\mathrm{BH}} = 4\pi G_N M^2$ [*[Derived]*, [18] Prop. 2.1; G_N from [16] Thm. 4.2]
- $S_{\mathrm{BH}}/S_{\mathrm{BH}}^{(\mathrm{Pl})} = G_N^{(1)}$ [*[Derived]*, [18] Cor. 2.3]

Class III (Spectral, depending on full $\mathrm{ZD}^*(\mathcal{A}_4)$ ***data).***

- *Two-loop* β*-function, IR zero* λ_{IR} [*[Derived]*, conditional, [17] Thm. 3.6]
- *Two-loop correction to* G_N^{-1}*, factor* $R \in [0.87, 0.95]$ [*[Derived]*, conditional, [17] Thm. 4.3]

Corollary 7.2 (*[Derived]*, [16] Lem. 3.1 + Theorem 7.1)**.** *Level-2 contributes zero to Class II and to all Class III spectral invariants. Level-2 belongs entirely to Class I: it is the* kinematic sector *of the spectrum, consistent with its quadruple protection of §3.*

Remark 7.3 (*[Derived-Negative]*). $\delta S_{\mathrm{GHY}} = 0$ (Class I) means the Gibbons–Hawking–York boundary term receives no algebraic IIB correction at tree level. The genuine quantum correction requires the Seeley–DeWitt boundary coefficient for S_{84} in the Black Circle background (OP 20.2, *[Open]*).

8. Quantitative holography

8.1. The three-step chain.

Proposition 8.1 (*[Derived]*, [1] Thm. 5.2; [2] Rem. 2.3, §2)**.** (Frobenius identity.) *For all* $a \in \mathcal{A}_n$*, all* $n \geq 0$*:* $\|L_a\|_F^2 = 2^n N(a)$*. At* $n = 4$*:* $\|L_a\|_F^2 = 16\, N(a)$*.*

Conjecture 8.2 (*[Conjecture, IIB-II]*; first explicit statement. Synthesises [1] Thm. 5.2, [2] §4.2 Connection 2, [16] Cor. 4.4, [18] Rem. 7.3).

$$S_{\mathrm{BH}} = \frac{\pi}{4} \cdot \|L_{C_0}\|_F^2 \cdot M^2.$$

The Bekenstein–Hawking entropy of the Black Circle is proportional to the squared Frobenius norm of the condensate left-multiplication operator.

Derivation. $S_{\mathrm{BH}} = 4\pi G_N M^2$ ([18] Cor. 2.3). $G_N \propto N(C_0)$ under IIB-II ([16] Cor. 4.4, with exact coefficient from Thm. 4.2). $\|L_{C_0}\|_F^2 = 16\, N(C_0)$ (Proposition 8.1 at $n = 4$). Substituting: $S_{\mathrm{BH}} = 4\pi G_N M^2 \propto \|L_{C_0}\|_F^2 \cdot M^2/16$, giving prefactor $\pi/4$. The conditionality is that of IIB-II. □

Remark 8.3. Paper 2, §4.2 ("Connection 2: Holography and the Bekenstein–Hawking entropy") identified this as a *structural analogy*, stating: "the analogy is structural, not quantitative." Paper 28, Rem. 7.3 converts it to a quantitative formula under IIB-II. Conjecture 8.2 records the conversion explicitly, for the first time, as a named result.

8.2. **Majorana entropic exclusion.** By Observation 3.3(3) and Conjecture 8.2, the Bekenstein–Hawking entropy counts only the **12 cokernel directions** of L_{C_0}. The 4 Majorana directions in $\ker(L_{C_0}) \cong V_{\mathrm{ni}}(C_0)$ are *entropically invisible.* The horizon entropy is a trace over 12 active (on-line Fano) directions, after removing the 4 non-incident ones.

9. The Black Circle as the logical mirror of $\mathcal{A}_3$

9.1. **The algebraic chain.** The following steps are all *[Derived]*, assembled from results of Papers 1–31:

1. The Fano plane governs multiplication in $\mathcal{A}_3$ and annihilation in $\mathcal{A}_4$: Theorem 2.3 [*[Derived]*, Papers 3, 4, 24, 29].
2. The kernel of L_a is isomorphic to the non-incidence subspace: Theorem 2.2 [*[Derived]*, Paper 29 Thm. 3.1].
3. The ZD structure of $\mathcal{A}_4$ generates G_N through the Sakharov–Adler supertrace, with effective spectral weight $\mathcal{S}_{\mathrm{eff}} = 164 = |\mathrm{GL}(3, \mathbb{F}_2)| - \dim\ker|_{n=4}$: [16] Thm. 4.2, Cor. 4.4; [19] Cor. 3.2 [*[Derived]*, conditional on Sakharov–Adler].
4. The Level-2 sector is simultaneously massless, gravitationally decoupled, gauge-invisible, and Majorana: Theorem 3.1 [*[Derived]*, conditional].
5. The holographic weight $\|L_{C_0}\|_F^2$ counts only the 12 active (non-kernel) directions: Conjecture 8.2 [*[Conjecture, IIB-II]*].

9.2. **The central conjecture.**

Conjecture 9.1 (*[Conjecture, IIB-I/II]*). Under IIB-I and IIB-II, the Black Circle is the *logical mirror* of $\mathcal{A}_3$: the algebraic boundary structure that provides $\mathcal{A}_3$ with its gravitational dynamics.

Precisely: (a) G_N — which is exclusively an $\mathcal{A}_4$ object, generated by the ZD structure via the annihilation face of the Fano bifrontality — is the gravitational constant that renders the matter of $\mathcal{A}_3$ dynamically consistent; (b) the Level-2 sector is dynamically transparent to gravity and to all gauge interactions, making it the kinematic sector of the spectrum; (c) the 4 Majorana modes of irr_8 are algebraically, gravitationally, and entropically isolated by a single Fano non-incidence structure.

Without G_N — which the algebra of $\mathcal{A}_3$ alone cannot produce — the matter of $\mathcal{A}_3$ has no gravitational field equations. The Black Circle is not an object *inside* $\mathcal{A}_4$: it is the condition for $\mathcal{A}_3$ to be physically complete.

Remark 9.2. The algebraic chain of §9.1 supports Conjecture 9.1 with three *[Derived]* steps (items 1–4) and one *[Conjecture, IIB-II]* step (item 5). The conjecture as a whole is *[Conjecture, IIB-I/II]*:

it requires IIB-I for the Born Rule boundary and IIB-II for the identification $G_N = N$. The Fano bifrontality (Theorem 2.3) is the algebraic core; the physical interpretation is the conjecture.

10. Open problems

OP 21.1 [*[Open]*, from [12]]. Do the SU(3) quantum numbers $\mathbf{8}\oplus\mathbf{6}$ of Level-2 under assumption (A2) admit a physical identification compatible with IIB-III?
OP 26.7 [*[Speculation]*, from [16]]. Do the gauge couplings g_1^2, g_2^2, g_3^2 emerge from N at CD levels $n = 1, 2, 3$ by the same Sakharov–Adler mechanism that gives G_N at $n = 4$?
OP 29.1 [*[Open]*, *[Conjecture,]*, from [19]]. Does there exist a compact Lie group G with $\dim G - \dim H = 14$ and $\mathrm{adj}(G)|_{\mathrm{PSL}(2,7)} \cong \mathrm{irr}_6 \oplus \mathrm{irr}_8$? ($G_2$ is excluded, [12] [*[Derived]*].)
OP 31.2 [*[Open]*, from [21]]. Does $H^{(a)}_{\mathrm{BC}} = 0$ impose an algebraic constraint on F_1/F_2 at QNM frequencies, potentially sharpening Conjecture 6.3 of [20]?
OP 32.1 [*[Open]*, **new**]. *What is the necessary and sufficient condition for the* irr_1 *singoletto to cross the CCC conformal boundary?* Under IIB-IV, Corollary 5.3 identifies irr_1 as a candidate messenger. Its three properties (massless, complementary-series boundary, S_4-symmetric) are necessary; sufficiency requires: (a) a precise formulation of the CCC crossing condition in terms of the conformal weight of a de Sitter scalar (boundary value problem at $r_s \to \infty$ in conformal time); (b) consistency with the Bunch–Davies boundary condition of Candidate (a) ([10]; [8] §3); (c) identification of the $\mathrm{irr}_{1'}$ exclusion ([12]) as a CCC crossing parity constraint. Tools: analytic (conformal weight computation); possibly GAP.

Summary table

Result	Label	Source
Fano bifrontality (Thm. 2.3)	*[Derived]*	Papers 3,4,24,29
$\ker(L_a) \cong V_{\mathrm{ni}}(a)$ (Thm. 2.2)	*[Derived]*	Paper 29 Thm. 3.1
Quadruple protection (Thm. 3.1)	*[Derived]*	Papers 11,19,20,21,26
Majorana threefold isolation (Obs. 3.3)	*[Observation]*/*[Conj., IIB-II]*	**New synthesis**
Fano partition $\mathrm{irr}_6 \oplus \mathrm{irr}_8$	*[Derived]*(module)	Paper 24
$\mathrm{irr}_6 \cong \mathrm{Sym}^2(\mathrm{irr}_3)$	*[Derived]*	Papers 21, 24
irr_1 singoletto (Rem. 5.2)	*[Derived]*(i–iii); *[Speculation, IIB-IV]*	Paper 21
Level-3 = 3×Level-2 (Prop. 6.1)	*[Derived]*	Papers 21, 22-B
Thermodynamic classes (Thm. 7.1)	*[Derived]*	Papers 26,27,28
Holography $S_{\mathrm{BH}} \propto \|L_{C_0}\|_F^2 \cdot M^2$	*[Conjecture, IIB-II]*	Papers 1,2,26,28
Black Circle as logical mirror (Conj. 9.1)	*[Conjecture, IIB-I/II]*	Synthesis
OP 32.1 (irr_1 CCC crossing)	*[Open]*	New

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