

THE MISSING 28: FANO NON-INCIDENCE AND THE OCTONION COCYCLE

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ABSTRACT. The $\binom{7}{3} = 35$ unordered triples of points in the Fano plane $\text{PG}(2, \mathbb{F}_2)$ split as $7_{\text{collinear}} + 28_{\text{non-collinear}}$. The 7 collinear triples (Fano lines) are the sector where the octonionic 2-cocycle c_{oct} satisfies the cocycle identity, where Z -type CSS stabilisers exist, and where the chirality elements $\Gamma_{prt} = L_{e_p} L_{e_r} L_{e_t}$ of the quaternionic subalgebras of $\mathbb{O}$ are well-defined. The 28 non-collinear triples are the obstruction sector: the cocycle fails, no Z -type stabiliser dual exists, and the Clifford representation of Paper 7 has no analogue.

We prove:

$$28 = \dim(\ker L_a)|_{n=4} \times |\text{PG}(2, \mathbb{F}_2)| = 4 \times 7,$$

where $\dim(\ker L_a) = 4$ is the Fano non-incidence count (Paper 29, Theorem 3.1). This is also the algebraic chain

$$28 = \frac{\dim \mathbb{O}}{2} \cdot (\dim \mathbb{O} - 1) = \binom{\dim \mathbb{O}}{2} = \dim(\mathfrak{so}(\mathbb{O})) = \dim(\mathfrak{so}(8)),$$

where the factors are $\dim(\ker L_a) = \dim \mathbb{O}/2$ (Paper 1, Theorem 5.17) and $|\text{Fano}| = \dim \mathbb{O} - 1$.

We additionally extract three analytic invariants of the ZD zeta function $Z(x)$ from Paper 3 never used in physics papers, assemble the 5-link chirality-element chain through which $R_1 = -7$ acquires its full structural meaning, identify the $[[15, 7, 3]]$ CSS bridge of Paper 8 as the Schubert stratum $G(3, 4; \mathbb{F}_2)$, state a conjecture relating the twisted c_{oct} cocycle to the non-split extension class of $\text{Aut}(\mathcal{A}_4)$ in $H^2(\text{PSL}(2, 7), (\mathbb{Z}/2\mathbb{Z})^4)$, and record a new observation: the number of (non-Fano triple, vertex) pairs equals $28 \times 3 = 84 = |\text{ZD}^*(\mathcal{A}_4)|$.

Epistemic key. *[Derived]* complete proof. *[Derived-Negative]* proves the negative. *[Observation]* algebraically established, no deeper categorical proof. *[Conjecture]* precise statement, gap identified. *[Speculation, IIB-II-strong]* structurally motivated. *[Open]* obstacle stated precisely.

1. INTRODUCTION: THE PARTITION $7 + 28 = 35$

The Fano plane $\text{PG}(2, \mathbb{F}_2)$ has 7 points and 7 lines. The $\binom{7}{3} = 35$ unordered triples of Fano points split naturally:

$$35 = 7_{\text{collinear}} + 28_{\text{non-collinear}}.$$

The 7 collinear triples are the Fano lines: the *good sector* where the octonionic 2-cocycle c_{oct} satisfies the cocycle identity (Paper 7 [6]), a well-defined Z -type CSS stabiliser dual exists (Paper 8 [7]; Steane $[[7, 1, 3]]$ code [18, 17]), and the chirality elements Γ_{prt} are well-behaved.

The 28 non-collinear triples are the *obstruction sector*: c_{oct} fails on them (its failure *is* the octonion associator), no Z -type stabiliser dual exists, and the Clifford representation has no analogue.

The number 28 appears in Papers 7 and 8 only as a complement count. Papers 29–34 independently derive $\dim(\ker L_a) = 4$ (Paper 29, Theorem 3.1) and use $|\text{PG}(2, \mathbb{F}_2)| = 7$ throughout. The product $4 \times 7 = 28$ is the core identity of this paper.

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2. ANALYTIC INVARIANTS OF $Z(x)$

Three results from Paper 3 [3] have never been exploited in the physics papers of the series.

Proposition 2.1 ([Derived], Paper 3 [3]).

$$Z(x) = 336 x^4 \cdot (x; 2)_4^{-1},$$

where $(x; q)_k = \prod_{j=0}^{k-1} (1 - q^j x)$ is the q -Pochhammer symbol with $q = 2$ (the Cayley–Dickson doubling base) and $k = 4$ (the number of Hurwitz algebras). $Z(x)$ is a q -hypergeometric function; the $q = 2$ specialisation connects ZD counting to the theory of q -series.

Proposition 2.2 ([Derived], Paper 3 [3]). For all $n \geq 4$:

$$|\text{ZD}(n)| = \frac{1}{4} \prod_{k=1}^3 (2^n - 2^k),$$

where $\prod_{k=1}^3 (2^n - 2^k)$ is a partial product of $|\text{GL}(n, \mathbb{F}_2)| = \prod_{j=0}^{n-1} (2^n - 2^j)$. At $n = 4$: $|\text{ZD}(4)| = |\text{GL}(4, \mathbb{F}_2)|/60 = 336$.

Proposition 2.3 ([Derived], Paper 3 [3]). The orbit count of degenerate ZD pairs satisfies $O_2(n) = |\text{ZD}(n-3)|/2$. The step 3 equals the number of non-trivial Hurwitz doublings $\mathcal{A}_1 \rightarrow \mathcal{A}_2 \rightarrow \mathcal{A}_3 \rightarrow \mathcal{A}_4$. ZD structure at level n contains an echo of ZD structure at level $n - 3$.

3. THE 28-TRIPLE IDENTITY

3.1. Main theorem.

Theorem 3.1 ([Derived]).

$$28 = \dim(\ker L_a)|_{n=4} \times |\text{PG}(2, \mathbb{F}_2)| = 4 \times 7.$$

Proof. By Paper 7 [6]: c_{oct} fails on exactly the non-collinear triples of $\text{PG}(2, \mathbb{F}_2)$. The total triple count is $\binom{7}{3} = 35$. Fano lines = 7. Non-Fano triples = $35 - 7 = 28$. By Paper 8 [7] Corollary (non-Fano obstruction): all 28 are confirmed as the CSS obstruction count. By Paper 29 [11] Theorem 3.1: $\dim(\ker L_a)|_{n=4} = 4$. By standard geometry: $|\text{PG}(2, \mathbb{F}_2)| = 7$. Product: $4 \times 7 = 28$. ✓ $\square$

Corollary 3.2 ([Derived]). $35 = (1 + \dim \ker L_a) \times |\text{PG}(2, \mathbb{F}_2)| = 5 \times 7$.

Corollary 3.3 ([Derived]). $7_{\text{Fano-compatible}} = m(\mu=-4) = s_1$: the CSS-compatible triple count equals the Level-1 spectral multiplicity and the normalized residue of $Z(x)$ at the $\mathbb{C}$ -pole (Papers 11, 33, 34 [9, 13, 14]).

Corollary 3.4 ([Derived]). $28 = \dim(\ker L_a) \times 7_{\text{CSS-compatible}}$: the CSS obstruction count is $\dim(\ker L_a)$ times the CSS-compatible count.

3.2. Algebraic chain to $\dim(\mathfrak{so}(8))$.

Proposition 3.5 ([Derived]).

$$28 = \frac{\dim \mathbb{O}}{2} \cdot (\dim \mathbb{O} - 1) = \binom{\dim \mathbb{O}}{2} = \dim(\mathfrak{so}(\mathbb{O})) = \dim(\mathfrak{so}(8)).$$

Proof. $\dim \mathbb{O} = 8$ (standard). $\dim(\ker L_a)|_{n=4} = 2^{n-2}|_{n=4} = 4 = \dim \mathbb{O}/2$ (Paper 1 [2] Theorem 5.17). $|\text{PG}(2, \mathbb{F}_2)| = 7 = \dim \mathbb{O} - 1 = |\text{Im}(\mathbb{O})|$ (standard). Therefore: $4 \times 7 = (\dim \mathbb{O}/2) \cdot (\dim \mathbb{O} - 1) = \binom{\dim \mathbb{O}}{2} = \dim(\mathfrak{so}(8))$. ✓ $\square$

Remark 3.6 ([Observation]). The identity $28 = \binom{\dim \mathbb{O}}{2} = \dim(\mathfrak{so}(8))$ has an independent combinatorial reading: the 28 generators of $\mathfrak{so}(8)$ label pairs from the 8-dimensional basis $\{e_0, e_1, \dots, e_7\}$ of $\mathbb{O} = \mathbb{R}^8$, which split into 7 pairs (e_0, e_i) (one per imaginary unit) and $\binom{7}{2} = 21$ pairs (e_i, e_j) with $i \neq j$ (one per pair of imaginary units, hence one per Fano-line pair since any two points in $\text{PG}(2, \mathbb{F}_2)$ lie on a unique Fano line) [16]. This labeling uses Fano *points* and *line-pairs*, not non-Fano triples.

Remark 3.7 ([Derived-Negative]). A canonical G_2 -equivariant bijection between the 28 generators of $\mathfrak{so}(8)$ and the 28 non-Fano triples does not exist. Under $G_2 \subset \text{SO}(7) \subset \text{SO}(8)$: $\mathfrak{so}(8) \cong \mathfrak{g}_2 \oplus V_7 \oplus V_7'$ (as G_2 -modules: adjoint + two copies of the fundamental 7), giving orbits of sizes $14 + 7 + 7$. The 28 non-Fano triples form a *single* orbit of size 28 under $\text{PSL}(2, 7)$ [15]: $|\text{PSL}(2, 7)|/|S_3| = 168/6 = 28$ (stabiliser $\cong S_3$, transitive on unordered bases of $\mathbb{F}_2^3$). The orbit structures ($14 + 7 + 7$ vs 28) are incompatible; no equivariant bijection exists.

3.3. Cross-connections.

Observation 3.8 ([Observation]). $28 = 2s_2 = 2 \dim G_2 = 4s_1$ [13, 14], where $s_1 = 7$, $s_2 = 14 = \dim G_2$, and $s_2 = 2s_1$ because $\dim(\text{SO}(7)/G_2) = 7 = |\text{Fano}|$ (Paper 33). The non-Fano triple count is connected to the $\mathbb{H}$ -residue of $Z(x)$.

Observation 3.9 ([Observation]). $84 = 28 \times 3 = |\text{ZD}^*(\mathcal{A}_4)|$. The number of pairs (non-Fano triple, distinguished vertex) equals $28 \times 3 = 84 = |\text{ZD}^*(\mathcal{A}_4)|$, the number of active sedenion zero-divisors [19]. Each non-Fano triangle has 3 vertices, and specifying a (triangle, vertex) pair gives a combinatorial set of size 84 that matches $|\text{ZD}^*(\mathcal{A}_4)|$.

Remark 3.10 ([Derived]). A sharper counting identity underlies Observation 3.9. Each Fano point $p \in \text{PG}(2, \mathbb{F}_2)$ belongs to $(28 \times 3)/7 = 12$ non-Fano triples (by PSL -transitivity of the 28-triangle set; 3 vertices per triangle, 7 points). Each ε -index class of $\text{ZD}^*(\mathcal{A}_4)$ contains $|\text{ZD}^*(\mathcal{A}_4)|/7 = 84/7 = 12$ active ZD elements (Paper 1 [2]). Thus: for each Fano point p , the number of non-Fano triples containing p equals the number of active ZD elements with ε -index p . This sharpens Observation 3.9 and makes the bijection question of OP 35.5 concrete at the level of individual Fano points.

Observation 3.11 ([Observation]). The partition $7 + 28 = 35$ is the numerical shadow of the Fano bifrontality (Paper 32 [12], Theorem 2.3):

$$35 = \underbrace{7}_{\text{incidence } (\mathcal{A}_3 \text{ mult.})} + \underbrace{28}_{\text{non-incidence } (\mathcal{A}_4 \text{ kernel})}.$$

Theorem 3.1 provides the missing factorization $28 = 4 \times 7$; together with Paper 32, this gives $35 = (1 + 4) \times 7 = 5 \times 7$.

4. THE CHIRALITY-ELEMENT CHAIN

Paper 7 [6] introduces $\Gamma_{prt} = L_{e_p} L_{e_r} L_{e_t}$, the chirality element of the quaternionic subalgebra $\mathbb{H}_{prt} = \text{Span}\{e_0, e_p, e_r, e_t\} \subset \mathbb{O}$, for each Fano line $\{p, r, t\}$. Paper 7 also notes: “the residue at each pole of $Z(x)$ corresponds to the count of Fano triples acting via Γ_{prt} .” This connection to $R_1 = -7$ has never been assembled into a chain.

Observation 4.1 ([Observation]). The following 5-link chain holds, with each link *[Derived]* in the cited paper:

$$\underbrace{7}_{\text{Fano lines (Paper 7)}} = \underbrace{7}_{\text{chirality elements } \Gamma_{prt} \text{ (Paper 7)}} = \underbrace{7}_{\text{Z-stabilisers Steane code (Paper 8)}} = \underbrace{m(\mu=-4)}_{\text{Level-1 mult. (Paper 11)}} = \underbrace{s_1}_{\text{residue of } Z(x) \text{ at } \mathbb{C}\text{-pole (Paper 33)}}.$$

The assembled chain is new; each link is *[Derived]* individually.

Remark 4.2. The complement: the 28 non-Fano triples $= \dim(\ker L_a) \times 7_{\text{chirality}}$ (Theorem 3.1) is the algebraic shadow of this chain. The chirality-compatible sector (7 elements) times the Fano non-incidence count (4) gives the chirality-incompatible sector (28).

5. THE $n=5$ CSS BRIDGE AS SCHUBERT STRATUM

Proposition 5.1 (*[Observation], [Derived] count*). *The 15 Fano planes appearing in the $[[15, 7, 3]]$ CSS bridge (Paper 8 [7] Theorem n5; Paper 5 [5]) are in bijection with the 15 elements of the Grassmannian $G(3, 4; \mathbb{F}_2)$ of 3-dimensional subspaces of $\mathbb{F}_2^4$, which is the Schubert stratum $k = n - 2 = 3$ of the Fano filtration of $\text{ZD}(5)$ (Paper 4 [4]).*

Count. $|G(3, 4; \mathbb{F}_2)| = \binom{4}{3}_2 = \binom{4}{1}_2 = (2^4 - 1)/(2^1 - 1) = 15$. ✓ *Paper 8 Theorem n5: the $n = 5$ CSS bridge uses exactly 15 Fano planes.* ✓

This bridges Papers 4 (Weil/Grassmannian), 5 (Reed–Muller), and 8 (CSS bridge) through the common Schubert geometry.

6. TWISTED COCYCLE AND THE NON-SPLIT EXTENSION

6.1. Setup. Left side (Paper 8 OP 2): does c_{oct} define a twisted 2-cocycle of $(\mathbb{Z}/2\mathbb{Z})^3$ with twisting data from the projective duality of $\text{PG}(2, \mathbb{F}_2)$? [7]

Right side (Papers 3, 9): Paper 3 [3] proves the non-split extension

$$1 \rightarrow (\mathbb{Z}/2\mathbb{Z})^4 \rightarrow \text{Aut}(\mathcal{A}_4) \rightarrow \text{GL}(3, \mathbb{F}_2) \rightarrow 1$$

for all $n \geq 4$. The extension class is non-trivial in $H^2(\text{PSL}(2, 7), (\mathbb{Z}/2\mathbb{Z})^4)$. Paper 9 [8] (Remark 3.4) determines: $H^2(C_2 \times \text{PSL}(2, 7), (C_2)^3) \cong \mathbb{F}_2^2$, extension class $(1, 0)$; the non-trivial PSL -component lives in $H^2(\text{PSL}(2, 7), (\mathbb{Z}/2\mathbb{Z})^3) \cong \mathbb{F}_2$.

6.2. Main conjecture.

Conjecture 6.1 (*[Conjecture]*, Papers 3, 7, 8, 9). The twisted 2-cocycle sought in Paper 8 OP 2 — the twisting data of c_{oct} under projective duality of $\text{PG}(2, \mathbb{F}_2)$ — is the component in $H^2(\text{PSL}(2, 7), V_{\text{lo}})$ of the non-split extension class of $\text{Aut}(\mathcal{A}_4)$, where the projection $(\pi_{\text{lo}})_*: H^2(\text{PSL}(2, 7), (\mathbb{Z}/2\mathbb{Z})^4) \rightarrow H^2(\text{PSL}(2, 7), V_{\text{lo}})$ uses the direct-sum decomposition $(\mathbb{Z}/2\mathbb{Z})^4 = V_{\text{lo}} \oplus V_{\text{hi}}$ (Paper 3 Remark, lo/hi bit components), and $V_{\text{lo}} = \langle \sigma_0, \sigma_1, \sigma_2 \rangle \cong (\mathbb{Z}/2\mathbb{Z})^3$ is generated by sign-flips on the Fano imaginary generators e_1, e_2, e_3 of $\mathcal{A}_3$. Both sides are classified by a single $\mathbb{F}_2$ -element.

Remark 6.2. The connection: V_{lo} is generated by the same sign-flips that determine c_{oct} ; both the twisting data and the extension class arise from non-associativity at the lo-bit level of $\mathcal{A}_4$.

OP 35.1 (*[Open]*). Prove Conjecture 6.1 by computing the restriction of the non-split extension class to V_{lo} and comparing with the c_{oct} -twisting class. GAP computation of $H^2(\text{PSL}(2, 7), (\mathbb{Z}/2\mathbb{Z})^4)$ is feasible (cf. Paper 9 [8] which computes the related group $H^2(C_2 \times \text{PSL}(2, 7), (C_2)^3)$).

7. $Z(G_{84}) = C_2$ AS ALGEBRAIC CPT SYMMETRY

Observation 7.1 (*[Observation]*, Paper 22-B [10]). $Z(G_{84}) = C_2 = \langle \pi \rangle$, where π is the global sign-reversal of all ZD elements: $\pi(e_p + \sigma e_{8+k}) = e_p - \sigma e_{8+k}$. π commutes with every ZD symmetry operation; G_{84} is transitive on $\{1, \dots, 84\}$; $|Z(G_{84})| = 2$.

Remark 7.2 (*[Observation]*, Paper 22-B [10]). $|N_{G_{84}}(G_{168})| = 336$: the normaliser of the order-168 subgroup of G_{84} has exactly 336 elements, which equals $|\text{ZD}(4)_{\text{dir}}|$ — the number of directed ZD pairs (Paper 3 [3]). Since $N_{G_{84}}(G_{168}) = \langle G_{168}, \pi \rangle = C_2 \times G_{168}$ (the central involution π normalises every subgroup), this is equivalent to $2|\text{PSL}(2, 7)| = 336$, confirming that the directed ZD count encodes the Fano symmetry group with both sign choices.

Remark 7.3 (*[Speculation, IIB-II-strong]*). Under IIB-II-strong [1], $\pi \in Z(G_{84})$ is the algebraic origin of CPT symmetry: the ZD structure is exactly symmetric under global sign inversion, which is the algebraic analog of charge conjugation C acting on S_{84} (the sedenion scalar field of Paper 11 [9]). The centrality $[Z(G_{84}), G_{84}] = 1$ is the algebraic counterpart of $[\text{CPT}, \text{Lorentz}] = 0$ in relativistic QFT.

8. OPEN PROBLEMS

OP 35.1 *[Open]*. Prove Conjecture 6.1: the c_{oct} twisting class equals the lo-bit projection of the non-split extension class.

OP 35.2 *[Open]*. Physical interpretation of $Z(G_{84}) = C_2$ (Observation 7.1) beyond the CPT reading: does π extend to a symmetry of the Black Circle geometry [9]?

OP 35.3 *[Open]*. q -hypergeometric interpretation of $Z(x)$ (Proposition 2.1): does $Z(x) = 336x^4 \cdot (x; 2)_4^{-1}$ belong to a known family of mock theta functions or Ramanujan q -series under $x = q^s$, $q = 2$?

OP 35.4 *[Open]*. Physical interpretation of the period-3 orbit recursion $O_2(n) = |\text{ZD}(n-3)|/2$ (Proposition 2.3). Does the period 3 equal the three Hurwitz doublings $\mathcal{A}_1 \rightarrow \mathcal{A}_2 \rightarrow \mathcal{A}_3 \rightarrow \mathcal{A}_4$ in a physically meaningful recurrence for the gravitational coupling $G_N^{(n)}$?

OP 35.5 *[Open]*, partially addressed. *Addressed*: $28 = \binom{\dim \mathfrak{o}}{2} = \dim(\mathfrak{so}(8))$ is *[Derived]* (Proposition 3.5). *Addressed*: a G_2 -equivariant bijection between the 28 generators of $\mathfrak{so}(8)$ and the 28 non-Fano triples is *[Derived-Negative]* (orbit incompatibility: $14+7+7$ vs 28 ; see also [16] where the $\mathfrak{so}(8)$ generators are labeled by Fano *points* and *line-pairs*, not non-Fano triples). *Remaining open*: does the numerical coincidence $|\{(\text{non-Fano triple}, \text{vertex})\}| = 84 = |\text{ZD}^*(\mathcal{A}_4)|$ (Observation 3.9) admit a canonical algebraic map?

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